Image formation and its physical basis

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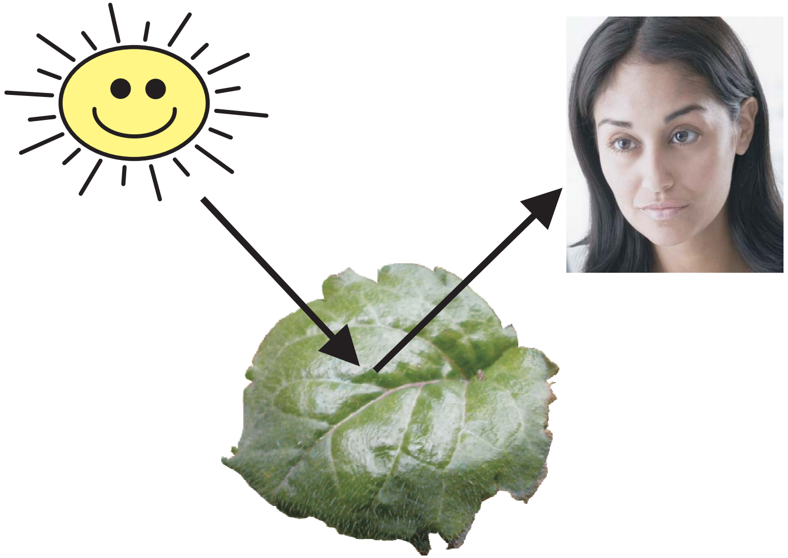
Outline of the lecture:

- Three reasons why to study image formation.
- Electromag. radiation. Its interaction with object surfaces.
- Radiometry, photometry, concepts.
- Irradiation equation.
- Surface reflectance. BRDF.
- Mathematical models of the reflectance.
- Lambertian surfaces.

Three reasons why to study image formation

Knowing geometric and radiometric relations between the 3D scene and its image allows the understanding:

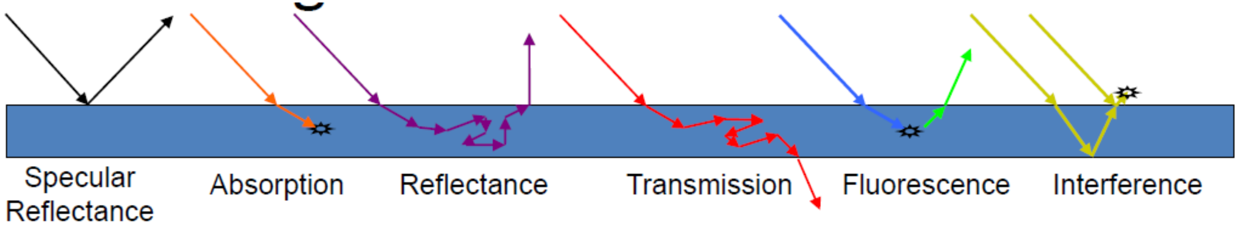
1. In computer graphics: How a real looking image can be rendered from a 3D model of the object/scene?
2. In computer vision: How to build machines that 'see'?
3. In computer vision: Why inverting image formation process is hopeless in most practically needed cases?



Three types of energy used in imaging

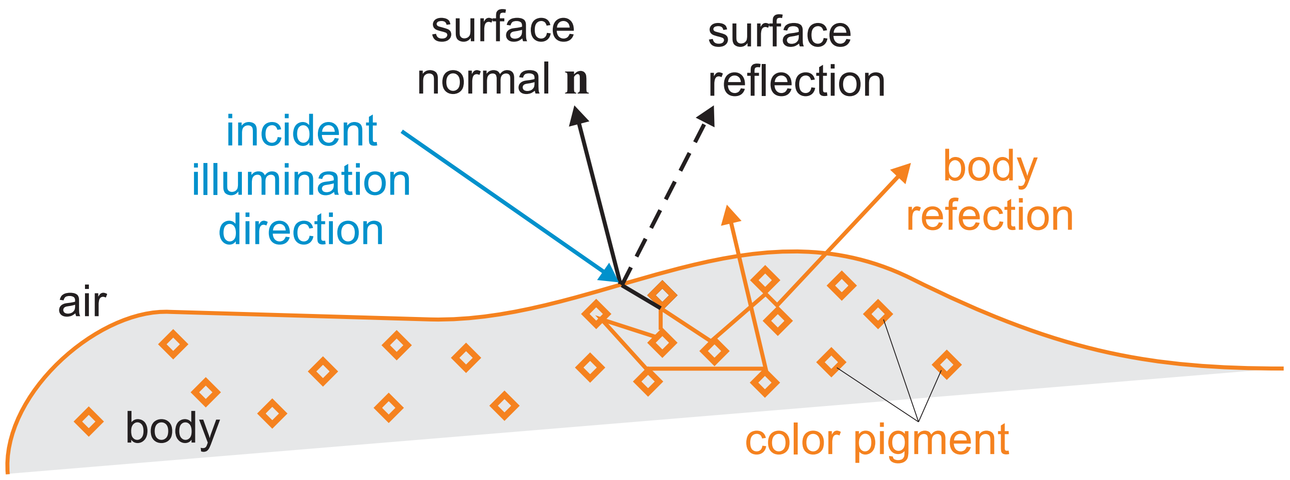
1. Electromagnetic radiation (will be developed in detail in the sequel). *Light is a "visually perceived radiant energy"*.
2. Radiation of elementary particles, e.g., electrons or neutrons.
3. Acoustic waves in gases, liquids and rigid bodies.
   In gases and liquids, only the longitudinal wave is propagated. In rigid bodies, also the transverse wave occurs. The speed of acoustic waves propagation is proportional to elastic properties of the medium through which it propagates.

- The radiation interacts with the matter either on its surface or in its volume.



- The radiation interacts with the matter either on its surface or in its volume. The radiation is emitted other due to the own heat motion of molecules (hot radiating body) or due to external stimulation (reflected radiation, luminescence, etc.).

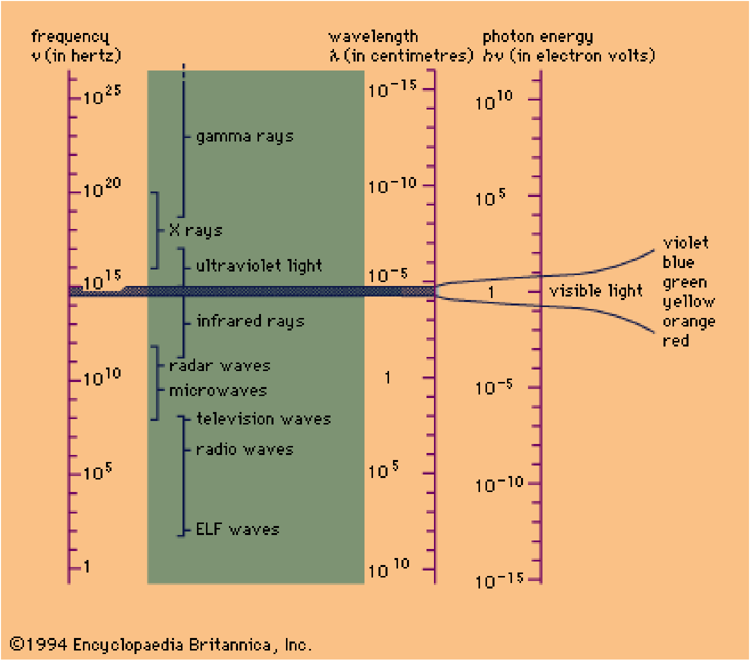
Body and surface reflection

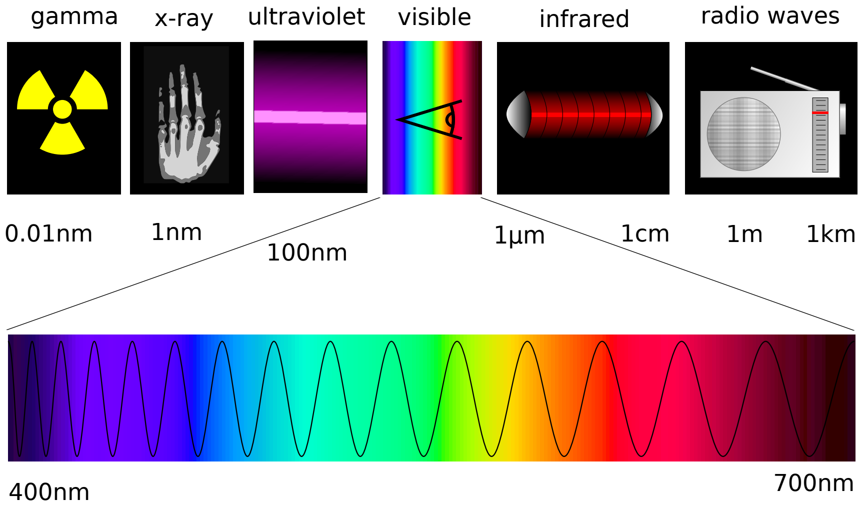


- Surface reflection – glossy, highlights are very directional. Called also specular reflection.
- Body reflection – absorption, diffused reflection to all direction, internal pigment absobrs part of the illuminant color spectra and causes color perception by humans.
- Metals manifest only the surface reflection.
- Dielectric materials (plastic, paint) manifest both surface and body reflection.

Electromagnetic radiation (1)

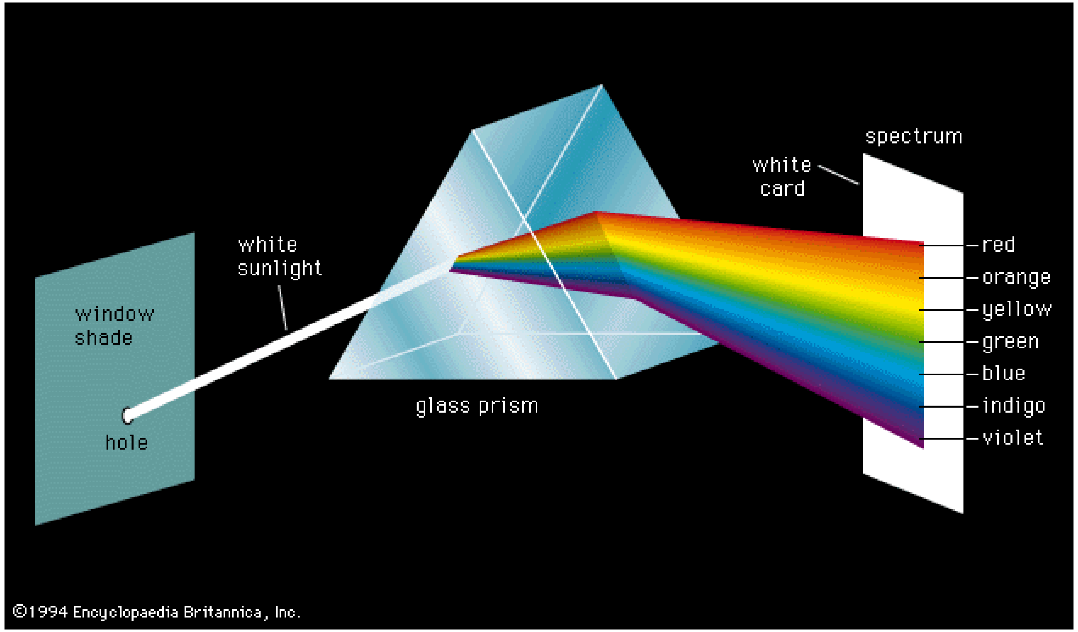
Includes γ rays, X rays, ultraviolet (UV) radiation, visible light, infrared radiation, microwaves and radiowaves.





Electromagnetic radiation (2)

- The radiation spreads in vacuum in the speed of light. If radiation penetrates the matter then its speed reduces and in addition it depends on the wavelength of the light.
- We will constrain our thoughts to the visible part of the electromagnetic radiation spectrum here. Isaac Newton used prism to show the spectrum in 1666.



Data in light useful in image analysis

1. Frequency of the radiation or alternatively its wavelength.
2. Amplitude, i.e. the light intensity.
3. Polarization mode for a transverse wave.
4. Phase of a wave which is accessible only if coherent imaging technology is used as in interferometry or holography.

To simplify the story, we will study formation of the image due to the radiation reflection from opaque surfaces (from a radiometric point of view).

Radiometry and photometry

Radiometry is a branch of physics studying the flow and the transmission of excited energy.

- Radiometry allows to explain image formation mechanism.
- Informally, the brightness in a pixel depends on a shape of the object, the reflectance properties of its surface, position of the viewer, positions and types of illumination sources.

Photometry, uses analogical quantities as radiometry but takes into account human perception system properties.

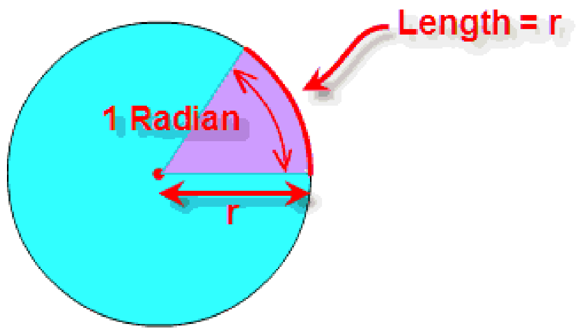
- Photometric quantities depend also on a spectral characteristics of the illumination source and the sensitivity of the photosensitive cells on the human eye retina.

Plane and spatial angles

Supplementary SI units [rad], [sr]

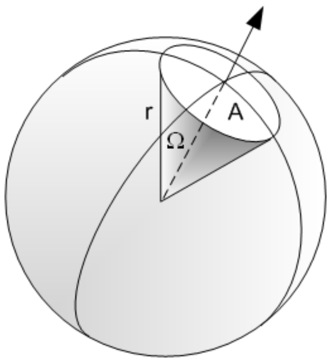
Plane angle – radian [rad].

- Radian is the plane angle between two radii of a circle, which cut off on the circle perimeter an arch equal in length to the radius.
- The arc in the full circumference extend has the length 2π [rad].



Spatial (solid) **angle** – steradian [sr].

- Steradian gives the spatial angle Ω of a cone with the vertex in the sphere center cutting off the sphere surface area A equal to the squared sphere radius.
- The spatial angle 4π [sr] equals to the entire sphere surface.



Emitting into the space

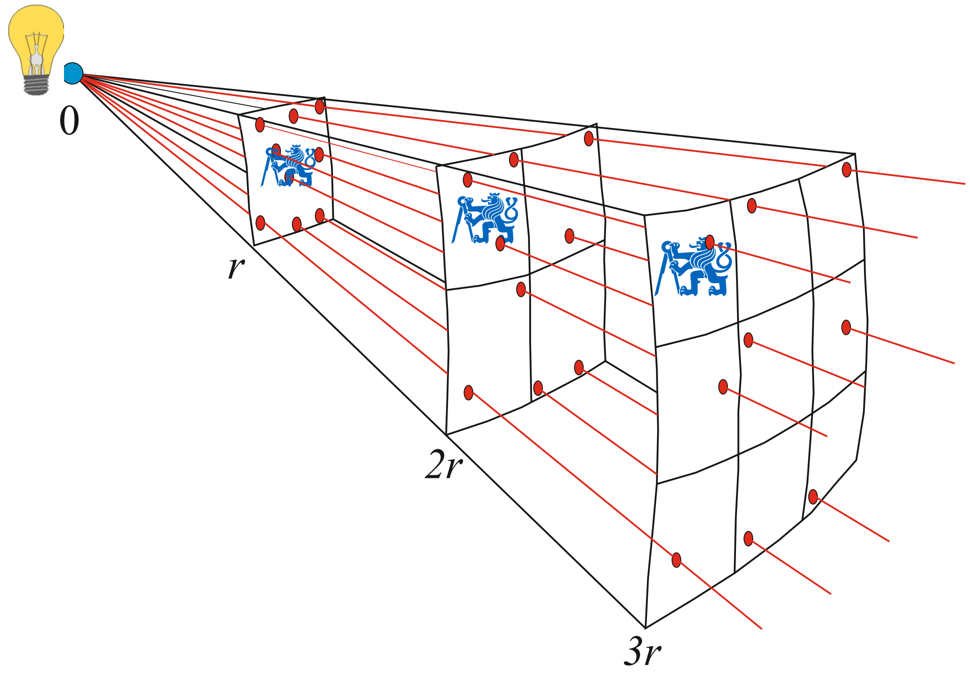
- The body surface can emit energy into the whole semi-sphere and, possibly, in different way in different directions.
- The spatial angle Ω is given by the area A on the surface of the unit sphere that is bounded by a cone with an apex in the center of the sphere.
- The whole half-sphere corresponds to the spatial angle of 2π [sr, (=steradian)].

  Q: Why?

  A: Because the surface area of the unit sphere (radius=1) equals to 4π.

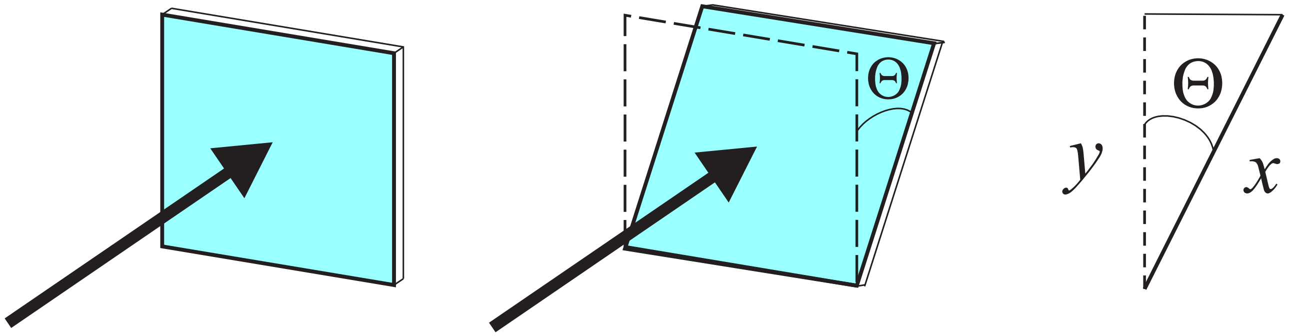
Intensity decrease, inverse square law

Consider a cone-shaped beam of light coming from a small point source and hitting a surface some distance away. The intensity is decreased by the square of the distance. See the figure for justification.



Cosine illuminance decrease of a slanted surface

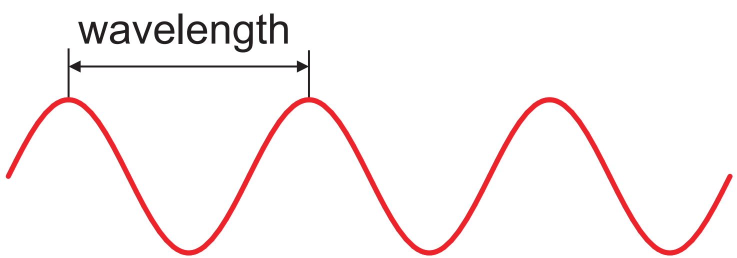
- The radiation effect is the strongest if the planar patch is perpendicular to incoming rays. If the planar patch is slanted then the radiation effect decreases.



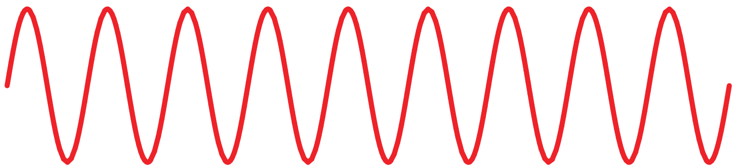
- $\cos\Theta = \frac{y}{x} \Rightarrow y = x\,\cos\Theta$.
- Similarly, if the original planar patch area was A then after slanting by the angle Θ then it is foreshortened as $A' = A\,\cos\Theta$.

Photons and waves

- Light behaves in some experiments as a particle (Isaac Newton, ~1670) and sometimes as a wave (Christian Huygens, ~1670).
- This discrepancy was solved by quantum mechanics (Max Planck, Albert Einstein) by introducing the concept of a photon.
- The photon can be imagined as a smallest energy portion (quantum) of certain frequency (wavelength) spreading in space (by the speed of light in the vacuum).
- The energy e of the photon is given by its frequency (wavelengths).



low energy, long wavelength



high energy, short wavelength

Photons, energy of the light, radiant flux

- Photons move in a straight path until it hit some obstacle, e.g. a surface of a solid with the surface area A.
- A photon on a solid surface can change a direction of its movement (e.g. reflect or diffuse) or perish (i.e. transform to another form of energy). However, the photon behavior on the surface is not of our concern for the moment.
- The light energy Q corresponds to the amount of photons, which hit the area A of the solid outer surface in a certain time interval.
- Radiant flux Φ $[W = Js^{-1}]$ definition assumes that the time interval is small in the limit

$$\Phi(A, t) = \frac{\mathrm{d}Q}{\mathrm{d}t}\,.$$

E.g. photons are counted for 1 second. Next we consider only 1 millisecond interval and get the energy $1000\ e$*, next 1 microsecond and* $10^6\ e$*, etc.*

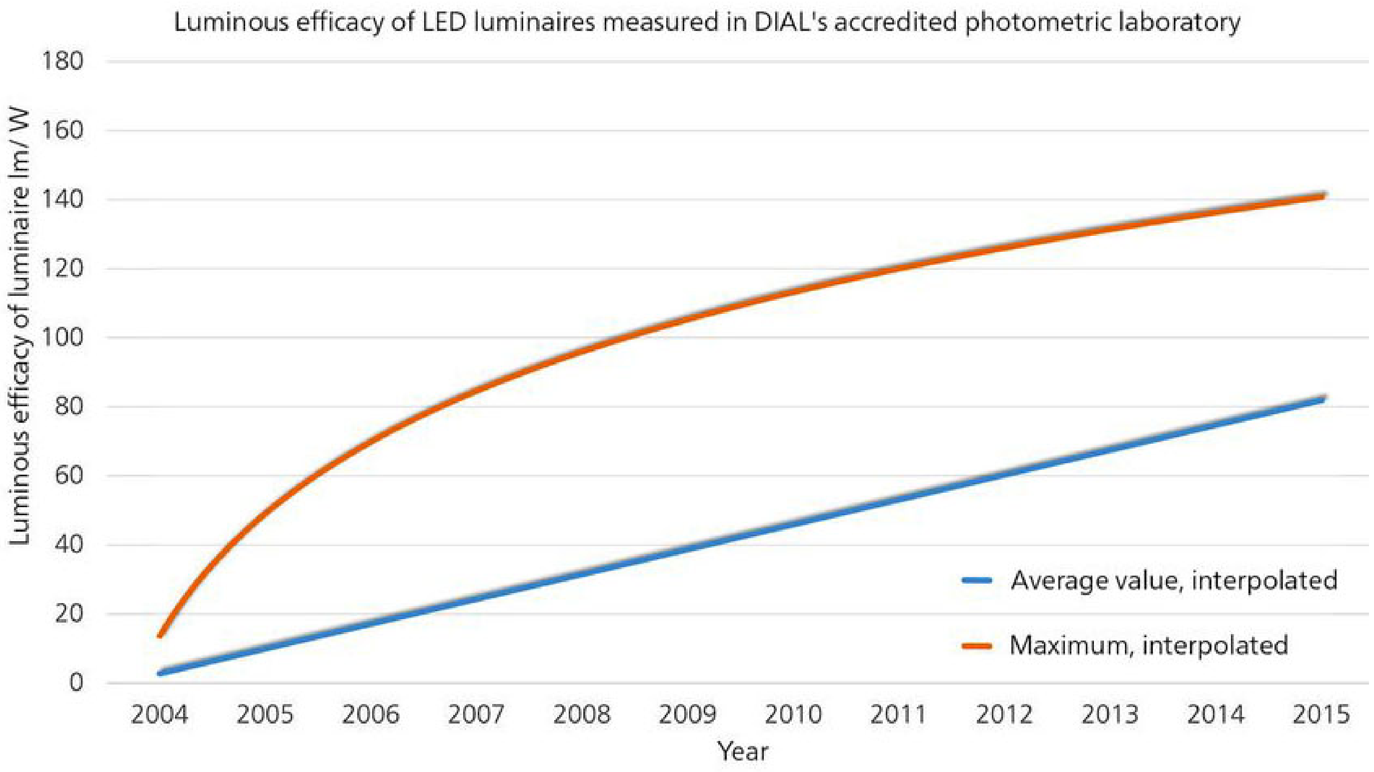
Concepts and quantities (1)

- Radiant flux Φ $[\mathrm{W} = Js^{-1}]$.
- Luminous flux Φ_{ph} $[\mathrm{lm}]$ (=lumen);
  $\mathrm{lm} = \mathrm{cd} \cdot \mathrm{sr}$ (candela times steradian);
  $1\ [\mathrm{W}] = 683\ [\mathrm{lm}]$ for the wavelength $\lambda = 555\ [\mathrm{nm}]$ (green) and the daylight vision in which retina cones are involved.
- Luminous efficacy $[\mathrm{lm/W}]$ tells how well a light source converts input energy into visible light, i.e. how well it converts input power in watts to lumens.
- Luminous efficiency [%] gives the relative value with respect to the maximum possible luminous efficacy, e.g. 683 $[\mathrm{lm/W}]$ for the ideal monochromatic 555 $[\mathrm{nm}]$ source.

Light sources luminous efficacy examples

Light source	Power [W]	Luminous flux [lm]	Luminous efficacy [lm/W]	Luminous efficiency [%]
candle	3.3	1	0.3	0.044
incandescent	100	1,750	18	2.6
tungsten halogen	100	1,880	19	2.8
LED, replaces 40 W	8	420	69	7.7
compact fluorescent	13	900	69	10.1
fluorescent	32	2,950	92	13.9
metal halide	175	14,000	80	11.7
high preasure sodium	150	16,000	107	15.6

LED light sources, luminous efficacy growth



See DIAL analysis for details and luminous efficacy theoretical limits for various light sources.

Luminous flux depends on wavelength λ

Let

- Let $K(\lambda)$ luminous efficacy $[\mathrm{lm\,W^{-1}}]$,
- $S(\lambda)$ $[\mathrm{W}]$ the spectral power of the light source,
- λ $[\mathrm{m}]$ is the wavelength.

Then, the luminous flux Φ_{ph} is proportional to the intensity of the human perception and is given by

$$\Phi_{ph} = \int_{\lambda} K(\lambda)\, S(\lambda)\, \mathrm{d}\lambda\,.$$

Concepts and quantities (2)

- Irradiance E $[\mathrm{W\,m^{-2}}]$ describes the power of the light energy that falls onto a unit area of the object surface, $E = \delta\Phi/\delta A$, where δA is an infinitesimal area element (patch) of the surface. E does not depend where the light is coming from.

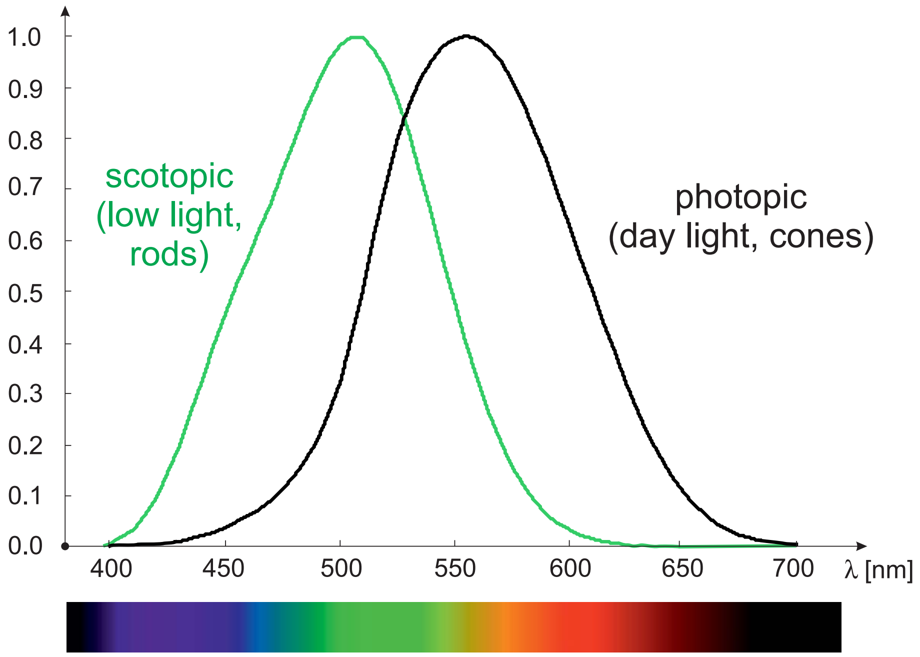
- The corresponding photometric quantity is the illumination E_{ph} $[\mathrm{lm\,m^{-2}}]$.
- Luminous intensity is a photometric quantity providing a measure of the wavelength-weighted power emitted by a light source in particular direction per spatial angle [cd (=candela, a SI base unit)].

The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 THz (555 nm, green) and that has a radiant intensity in that direction of 1/683 watt per steradian.

A luminous intensity of a candle is approximately 1 cd which is the source of candela name.

Luminosity function

- Luminosity function describes the average spectral sensitivity of human visual perception to brightness. It is based on subjective judgements of which of a pair of different-colored lights is brighter, to describe relative sensitivity to light of different wavelengths.
- It was established by the Commission Internationale de l'Éclairage (CIE) in 1931. Newer improvements exist.



Luminocity function normalized by maximum.

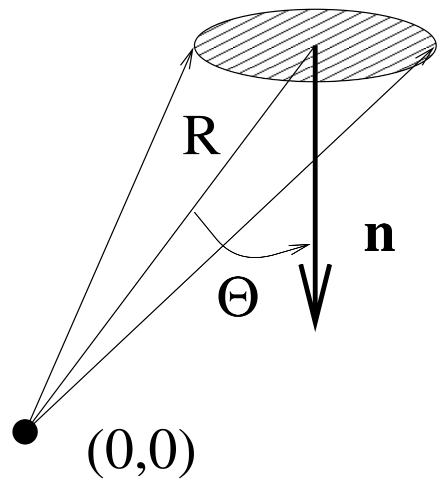
Concepts and quantities (3)

- The much used photometric quantity in the image analysis is the luminance L_{ph} (informally brightness) $[\mathrm{lm\,m^{-2}\,sr^{-1}}]$ because it well represents the quantity measured by the camera. is the power of light that is emitted from a unit surface area into the unit spatial angle $[\mathrm{sr}]$.
- The radiometric equivalent of the brightness is the radiance L $[\mathrm{W\,m^{-2}\,sr^{-1}}]$, which which is the power of light that is emitted from a unit surface area into the unit spatial angle $[\mathrm{sr}]$.

Relative foreshortening of a slanted surface patch area (reminder)

A small surface patch A at the distance R from the origin, i.e. $R^2 \gg A$, with the angle Θ between the normal vector to the patch and the radius vector between the origin and the patch corresponds to the spatial angle Ω,

$$\Omega = \frac{A \cos \Theta}{R^2} .$$

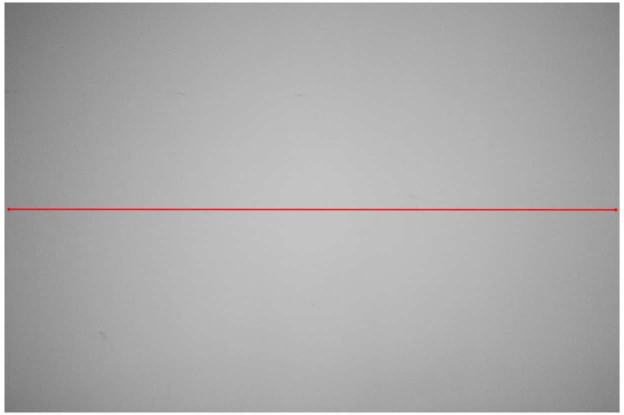


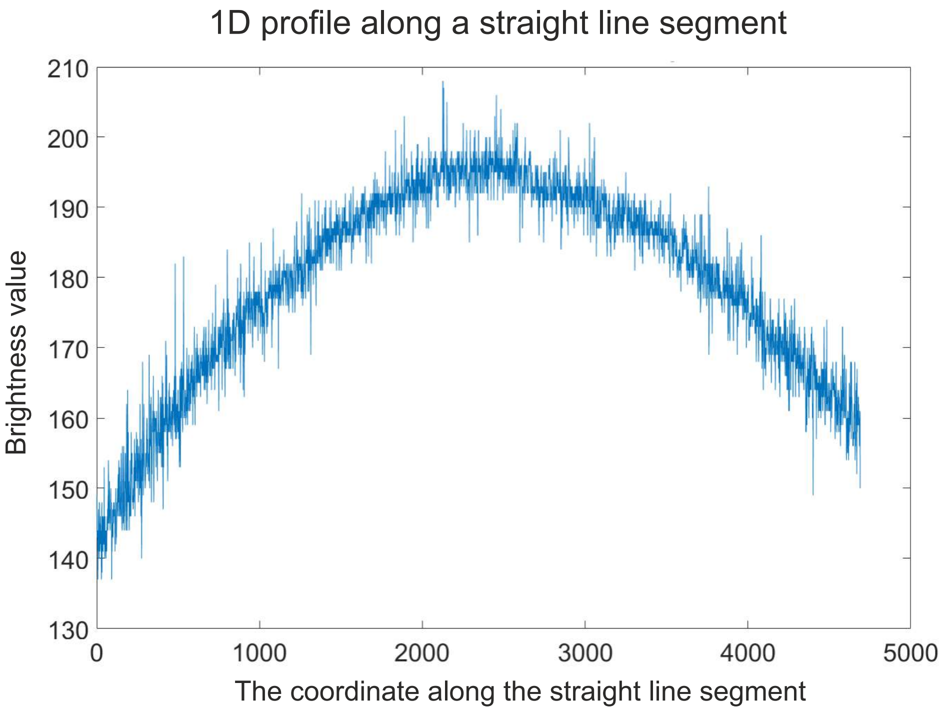
Natural vignetting

- Natural vignetting is a systematic error (abberation).
  (Note: The optical and mechanical vignetting exist too. To be discussed in this lecture later).
- Natural vignetting describes the phenomenon that rays with a bigger angle α (i.e. more inclined from the optical axis) are attenuated more. The attenuation is given by the factor $\cos^4 \alpha$. We will derive it shortly in the irradiance equation.
- This implies that this abberation is more pronounced with wide-angle lenses than with tele-lenses.
- As the natural vignetting is a systematic error, it can be compensated, of course, only for a camera with fixed focus and radiometrically calibrated setup.
- We illustrate the natural vignetting on the example first and derive it (irradiance equation) next.

Natural vignetting, practical example

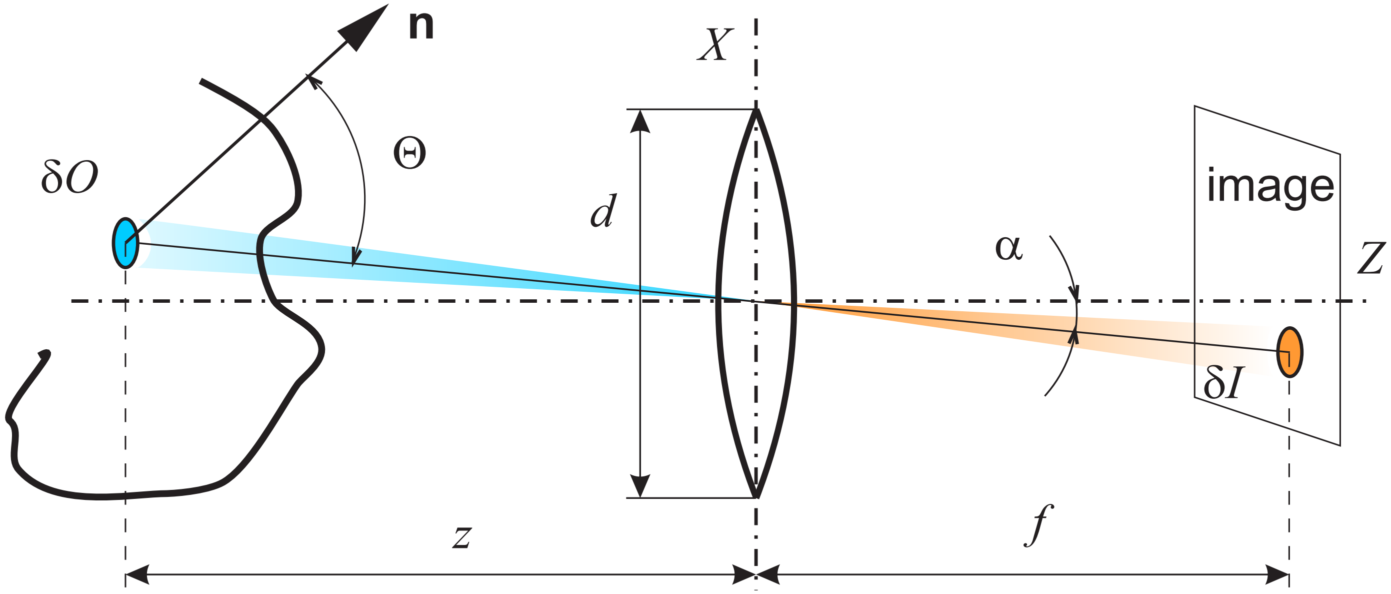
A sheet of white paper illuminated by natural diffused light was captured using a decent fix focal length lens Canon EF 50 mm f/1.8 II with maximum aperture 1:1.8.





Irradiance equation (1)

We shall consider the relationship between the irradiance E measured in the image and the radiance L produced by a small patch on the object surface. Only part of this radiance is captured by the lens of the camera.



Let consider a pinhole camera model first.

Irradiance equation (2)

- A ray passing the center of the lens does not diffract. Thus the spatial angle matching to the elementary patch in the scene equals to the spatial angle matching to the elementary area in the image.
- The slanted area δI as seen from the center of the coordinate system is $\delta I \cos\alpha$. Its distance from the center of the optical system is $f/\cos\alpha$.
- The corresponding spatial angle is

$$\frac{\delta I \cos\alpha}{\left(\frac{f}{\cos\alpha}\right)^2} .$$

- Analogously, the spatial angle corresponding to the elementary patch δO on the object surface is

$$\frac{\delta O \cos\Theta}{\left(\frac{z}{\cos\alpha}\right)^2} .$$

Irradiance equation (3)

As the spatial angles on the surface and image sides are equal

$$\frac{\delta O \cos\Theta}{\left(\frac{z}{\cos\alpha}\right)^2} = \frac{\delta I \cos\alpha}{\left(\frac{f}{\cos\alpha}\right)^2}$$

$$\frac{\delta O \cos\Theta}{\frac{z^2}{\cos^2\alpha}} = \frac{\delta I \cos\alpha}{\frac{f^2}{\cos^2\alpha}}$$

$$\frac{\delta O \cos\Theta}{z^2} = \frac{\delta I \cos\alpha}{f^2}$$

$$\frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\Theta} \cdot \frac{z^2}{f^2}$$

Irradiance equation (4)

Consider how much light energy passes through the lens if its aperture has the diameter d.

The spatial angle Ω_L that sees the lens from the elementary patch on the object is

$$\Omega_L = \frac{\pi}{4} \frac{d^2 \cos\alpha}{\left(\frac{z}{\cos\alpha}\right)^2} = \frac{\pi}{4} \left(\frac{d}{z}\right)^2 \cos^3\alpha \,.$$

Let L be the radiance of the object surface patch that is oriented towards the lens. Then the elementary contribution to the radiant flux Φ $[W]$ falling at the lens is

$$\delta\Phi = L\,\delta O\;\;\Omega_L\;\cos\Theta = \pi\,L\,\delta O\,\left(\frac{d}{z}\right)^2 \frac{\cos^3\alpha\cos\Theta}{4}\,.$$

Irradiance equation (5)

The lens concentrates the light energy into the image. If energy losses in the lens are neglected and no other light falls on the image element, we can express the irradiation E of the elementary image patch as

$$E = \frac{\delta\Phi}{\delta I} = L\,\frac{\delta O}{\delta I}\,\frac{\pi}{4}\left(\frac{d}{z}\right)^2 \cos^3\alpha\,\cos\Theta\,.$$

If we substitute for $\frac{\delta O}{\delta I}$, we obtain an important equation that explains how scene radiance L influences irradiance E in the image

$$E = L\,\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha\,.$$

$n_f = \frac{f}{d}$ is the f-number of a lens

- In the irradiance equation,

$$E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha \,.$$

  the term $\frac{d}{f}$ appeared.

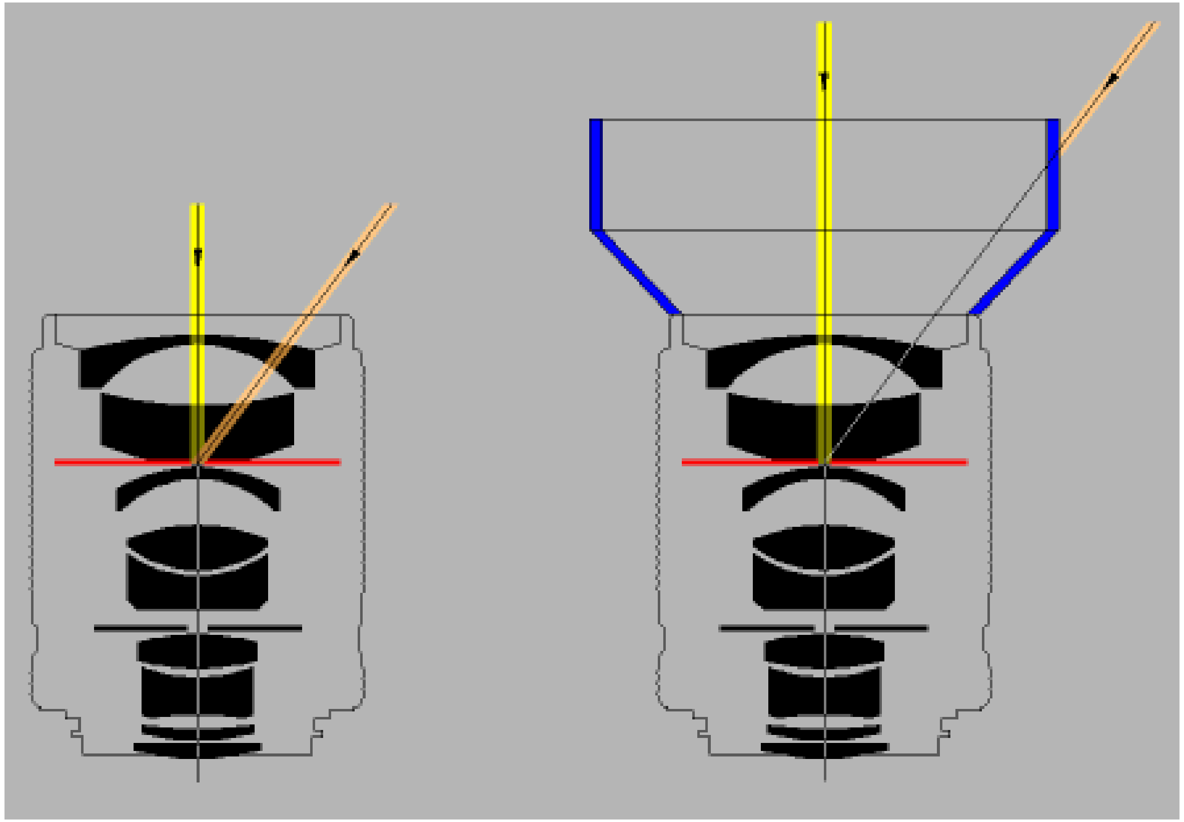
- Its inverted value $n_f = \frac{f}{d}$ is called f-number of the lens.
- f-number n_f is an important parameter characterizing the lens. It tells how much the lens differs from a pinhole camera.

Optical vignetting

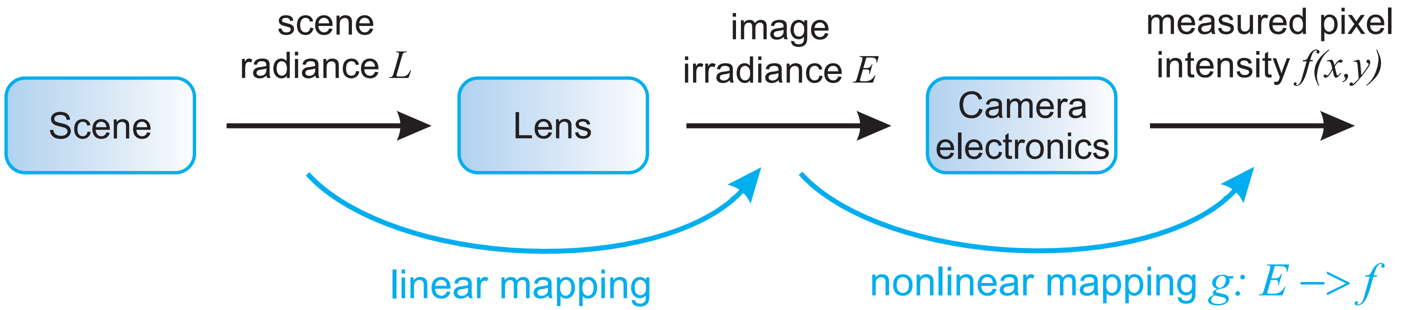
- Real lenses are composed of several simple lenses and are several centimeters wide. Because of that not all diaphragm could be available for all rays.
- The phenomenon is naturally more pronounced for the open diaphragm.

Mechanical vignetting

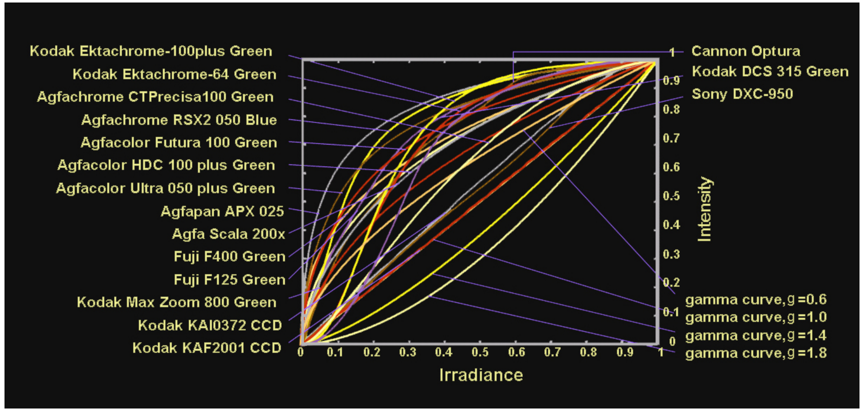
It concerns only careless users.



Relation between the scene radiance L and the pixel intensity $f(x, y)$

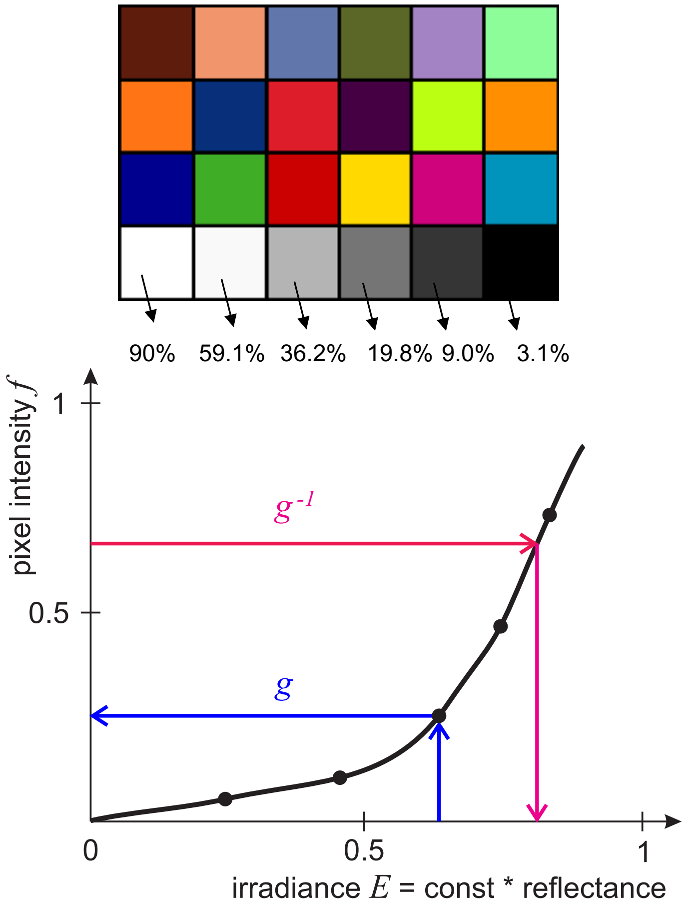


Nonlinear mapping in 'camera electronics' studied in M.D. Grossberg and S.K. Nayar: Modeling the Space of Camera Response Functions, IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol.26, No.10, Oct 2004, pp. 1272-1282.



Radiometric calibration

- Important for 'measuring' applications as photometric stereo, de-weathering, image-based rendering.
- The inverse mapping $g^{-1}: f \rightarrow E$ is established experimentally. The unique inverse exists due to the monotonicity of g.
- Calibration gray/color chart with known reflectances is used.
- Multiple camera exposures for varying image intensities are used.



High dynamic range, formulation

Varying illuminance [lux = lumen / m^2; symbol lx] = luminous flux per unit area.

Direct sun	100,000 lux
Sunny day	50,000 lux
Cloudy day	5,000 lux
Office	400 lux
Home lighting	10 lux
Street lamps	1 lux
Full moon	0.1 lux
Quarter moon	0.01 lux
Clear moonless night	0.001 lux
Cloudy moonless night	0.0001 lux

Total lighting range

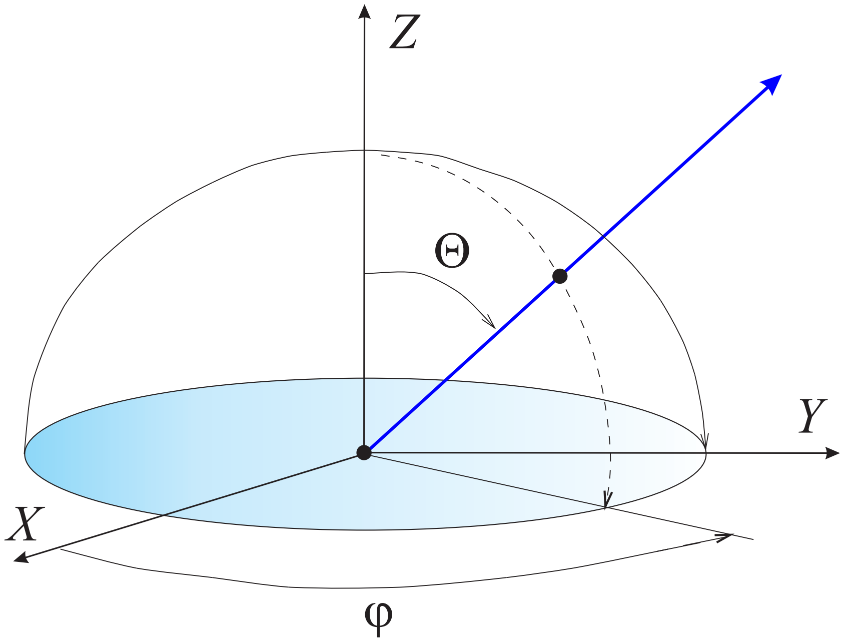
Camera sensor range

High dynamic range photography – example

Courtesy: photo Whitson Gordon

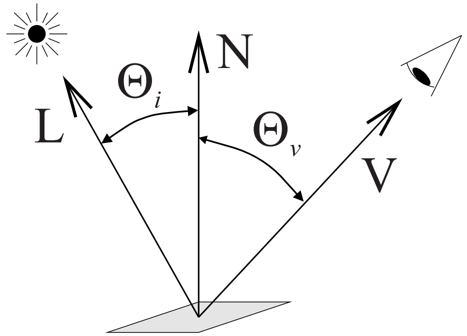
Directions given by the azimuth and elevation angles

- We will need to specify the directions in 3D space. They will be given by the direction of a vector.
- The vector direction will be given by the azimuth ϕ and the elevation angle Θ similarly as in spherical coordinates.



Radiance observed by a viewer

- We assume an infinitesimally small patch on a non-transparent object surface. The patch orientation is given by its normal vector $\mathbf{N}$.
- The patch is lit by the illuminant providing the radiance $L(\Theta_i, \phi_i)$. It causes irradiance $E(\Theta_i, \phi_i)$ at the patch.
- If the surface patch does not emit its own energy, the viewer 'sees' the radiance $L(\Theta_v, \phi_v)$.
- The radiance $L(\Theta_v, \phi_v)$ depends on the direction vector $\mathbf{L}$ to the light source (the simplest case, a point source, is considered), the direction vector $\mathbf{V}$ towards the viewer, the patch orientation $\mathbf{N}$ of the surface patch, and the surface reflectance f_r of the patch.



- $\mathbf{L}$ from Light (direction to)
- $\mathbf{N}$ from Normal vector
- $\mathbf{V}$ from Viewer (direction to)

Reflectance f_r at a small surface patch

- We assume an infinitesimally small patch of a non-transparent object. The patch orientation is given by its normal vector $\mathbf{N}$.
- We assume that the surface patch does not emit its own energy.,
- The patch is lit by the illuminant providing the irradiance at the surface path $E(\Theta_i, \phi_i)$.
- The incoming energy causes that the surface patch emits the radiance into the direction of the viewer $L(\Theta_v, \phi_v)$.
- The radiance L of the surface patch depends proportionally on the incoming irradiation E. The proportionality is given by the surface reflectance f_r, i.e. the ability of the surface patch to convert the incoming irradiation $E(\Theta_i, \phi_i)$ into the radiation $L(\Theta_v, \phi_v)$.

BRDF

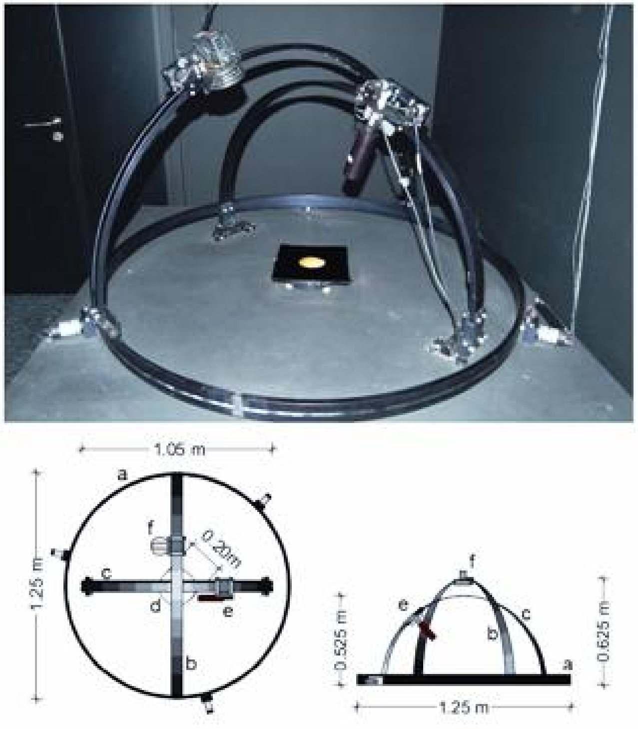
- BRDF – Bidirectional Reflectance Distribution Function f_r.
- BRDF describes the brightness of an elementary surface patch for a specific material, a light source, and viewer directions as a ratio between a measured radiance reflected from a surface caused by irradiance E from a certain direction. The influence of the phase is neglected for simplicity.

$$f_r(\Theta_i, \varphi_i; \Theta_v, \varphi_v) = \frac{\delta L(\Theta_v, \varphi_v)}{\delta E(\Theta_i, \varphi_i)} \quad [\mathrm{sr}^{-1}] \, .$$

- BRDF is also important for realistic rendering in computer graphics. It is needed in its full complexity used for modeling reflection properties of materials with oriented microstructure (e.g., tiger's eye—a semi-precious golden-brown stone, a peacock's feather, a rough cut of aluminum).
- Helmholtz's reciprocity, which follows from 2nd Law of Thermodynamics

$$f_r(\Theta_i, \varphi_i; \Theta_v, \varphi_v) = f_r(\Theta_v, \varphi_v; \Theta_i, \varphi_i) \, .$$

Measuring BRDF on a goniometer

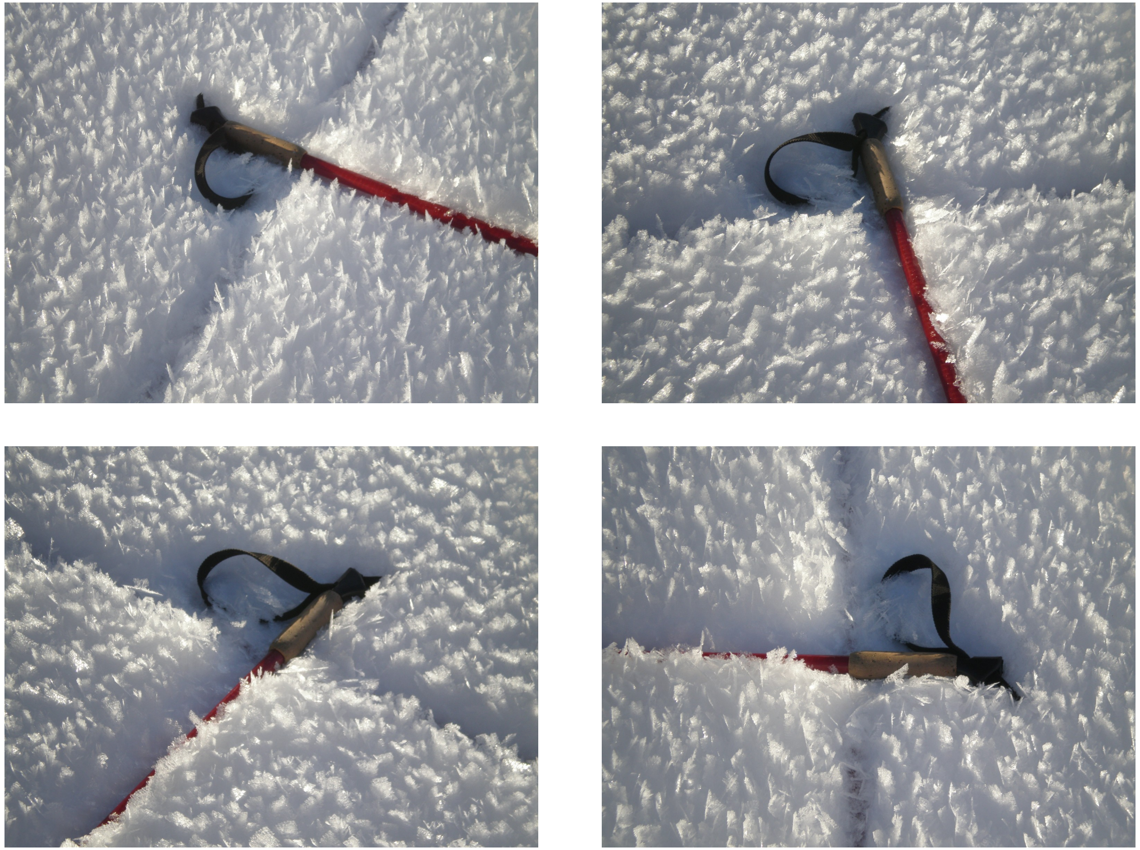


Source: Dimitrios Biliouris et al.: RPV Model Parameters Based on Hyperspectral Bidirectional Reflectance Measurementsof Fagus sylvatica L. Leaves. Remote Sens. 2009, 1(2), 92-106; doi:10.3390/rs1020092

Example of the complex BRDF – the frozen snow

The frozen snow, detail

The frozen snow, dependence on the orientation



A simplified BRDF

- Many practically important surfaces are isotropic, i.e. their BRDF does not depend on the rotation of the elementary surface patch along the normal to the surface. In such a case, f_r depends only on the absolute difference of azimuths of the directions to the light source and the viewer, $\varphi_i - \varphi_v$, which means on $f_r(\Theta_i, \Theta_v, |\varphi_i - \varphi_v|)$.
- The simplification holds for opaque (Lambertian) surfaces, ideally reflecting surfaces (mirrors) and their combinations.

Reflectance coefficient = albedo

- The albedo expresses what ratio of the incident energy the surface reflects back to the halfspace.
- Simplification: We will neglect the influence of a surface color and the dependence of the albedo on the light wavelength λ.
- $E_i(\lambda)$ is the radiance caused by the illuminated surface patch and $E_r(\lambda)$ is the flow of energy on the unit area radiated by the surface patch back to the halfspace.
- The sought ratio is the integral of the radiance L from the surface patch in a spatial angle Ω, which matches the whole halfspace.

$$E_r = \int_\Omega L \, \mathrm{d}\,\Omega \,.$$

Reflectance function $\boldsymbol{R(\Omega)}$

- $R(\Omega)$ models the influence of the local changes of the surface geometry to the dissipation of the reflected energy into the space.
- Let Ω be the infinitesimal space angle around the view direction,

$$\int_\Omega R \, \mathrm{d}\Omega = 1 \, .$$

- The reflective properties of the surface depend on three angles in general, which express mutual between the direction towards the illumination source $\mathbf{L}$, the direction towards the viewer $\mathbf{V}$, and the local orientation of the surface patch given by its normal vector $\mathbf{n}$.

Reflectance function (2)

- Cosines of vectors (directions) towards the illumination source $\mathbf{L}$, towards the viewer $\mathbf{V}$ and local orientation of the surface given by the normal vector $\mathbf{n}$ can be rewritten as the dot product, denoted as $(\,\cdot\,)$, of vectors.
- Then, the reflectance function $R\,(\mathbf{n}\cdot\mathbf{L}, \mathbf{n}\cdot\mathbf{V}, \mathbf{V}\cdot\mathbf{L})$.

A special case = Lambertian surface

- Lambertian surface (also ideally opaque surface, ideal diffusion surface) reflects the light energy uniformly into all directions.
- That is the reason why the radiance (and also the perceived brightness) is the same from all the directions,

$$f_{\text{Lambert}}\left(\Theta_i, \Theta_v, \varphi_i - \varphi_v\right) = \frac{\rho(\lambda)}{\pi}\,.$$

- The name comes from the book Photometria by Johann Heinrich Lambert issued in Latin in 1760. The word "albedo" was used there for the first time.

Lambertian surface (2)

- The reflectance of the Lambertian surface can be expressed as a cosine law for the constant albedo $\rho(\lambda)$

$$R = \frac{1}{\pi}\,\mathbf{n}\cdot\mathbf{L} = \frac{1}{\pi}\,\cos\Theta_i\ .$$

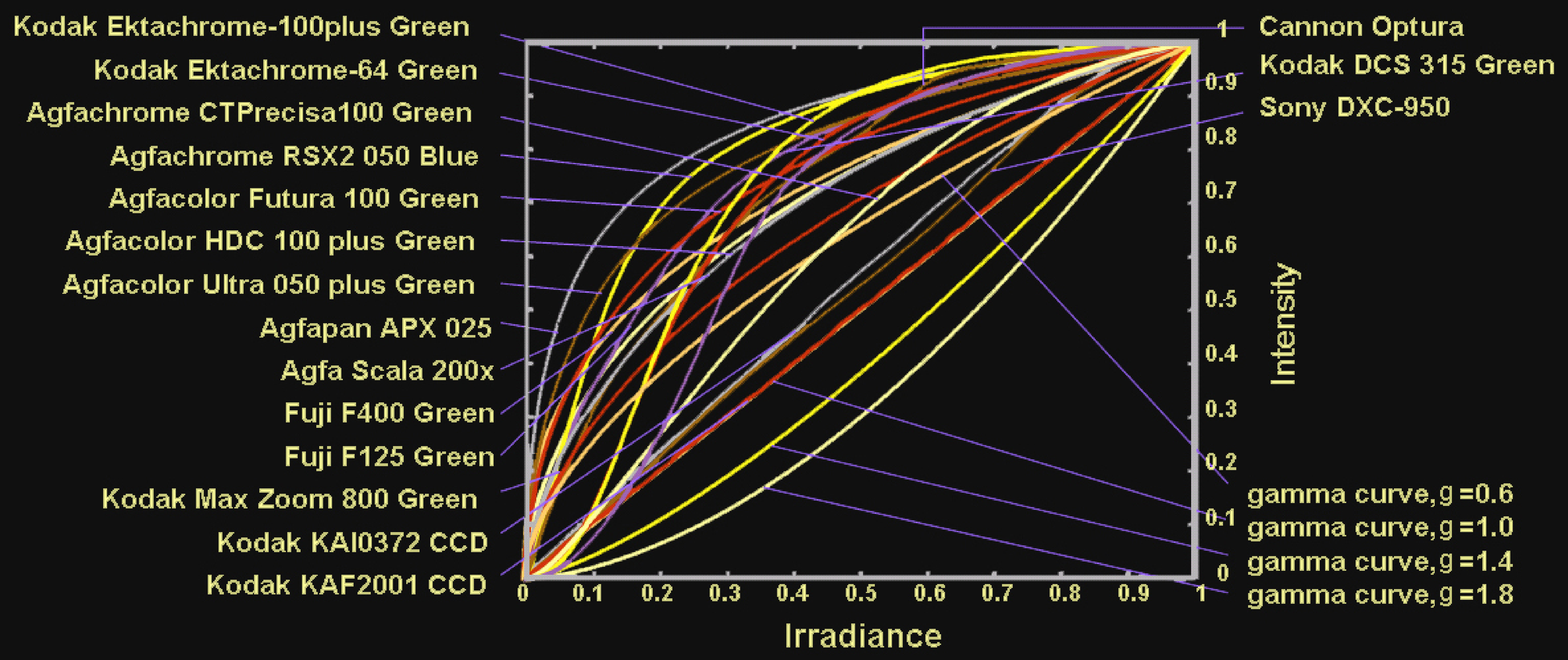
- Notice that the reflectance function ρ does not depend on the viewing direction $\mathbf{V}$.
- Lambertian reflectance model is very popular because of its simplicity.

Numerical reflectance values for Lambertian surfaces

- Lambertian surface illuminated by parallel light rays with the polar angle Θ and illumination E.
- The radiance L is observed.
- Examples of materials their reflectance can be approximated as Lambertian for values $\rho(\lambda)$, where λ lies in approximately in the middle of the visible spectrum.
- White blotting paper $=0.8$. White paper $=0.68$. White ceiling or yellowish paper $=0.6$. Dark brown paper $=0.14$. Dark velvet $=0.004$.

Ideal mirror surface

- It reflects the irradiance from the direction (Θ_i, φ_i) into the direction $(\Theta_i, \varphi_i + \pi)$.
- The actual surface is not visible. The virtual image is shown which is mirror reflected image of the illumination sources, i.e. surfaces in the scene emitting light.



Kodak Ektachrome-100plus Green
Kodak Ektachrome-64 Green
Agfachrome CTPrecisa100 Green
Agfachrome RSX2 050 Blue
Agfacolor Futura 100 Green
Agfacolor HDC 100 plus Green
Agfacolor Ultra 050 plus Green
Agfapan APX 025
Agfa Scala 200x
Fuji F400 Green
Fuji F125 Green
Kodak Max Zoom 800 Green
Kodak KAI0372 CCD
Kodak KAF2001 CCD
Cannon Optura
Kodak DCS 315 Green
Sony DXC-950
gamma curve,g =0.6
gamma curve,g =1.0
gamma curve,g =1.4
gamma curve,g =1.8
Intensity
Irradiance
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1