Geometric optics and camera, practically

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Outline of the talk:

- Motivation, lens of a camera.
- Why are lenses needed?
- Geometric optics as a simplified model.
- Lens from a physical point of view.

- Depth of focus. Depth of field.
- Lenses.
- Lenses abberations.
- Camera, radiometric image formation.

Entire image acquisition chain, overview

A look at an entire chain: from the observed property of interest through radiance L and irradiance E to an electrical signal and finally to a digital image.



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Two image acquisition options:

- Direct observation there is one-to-one correspondence between a point in a 3D scene and its 2D image (e.g., a ray in the projective transformation).
- Indirect observation provides also a spatially dependent radiance L but there is no one-to-one correspondence between 3D and 2D information (e.g., radar, tomography, spectral imaging techniques, magnetic resonance).

Single-lens reflex camera, a cross-section





Basic elements of a camera



- Photographic lens. Fixed or varying focal length. Diaphragm with varying aperture. Composed from several optical lenses, single optical elements.
- **Shutter**. Mechanic or electronic.
- Optical viewer (absent with cheaper cameras).
- Matrix of light sensitive sensors. CCD or CMOS.
- Amplifier modifying the signal from sensors.

Analog/digital converter.

- Computer converting raw image data to a viewable representation.
- LCD display for viewing images. LCD stands for Liquid Crystal Display.
- **Memory** medium, often removable.
- Energy source, battery or rechargeable battery.

Improving parts/functions of the camera



- Automatic exposition setting, i.e. joint setting of the diaphragm opening and shutter speed.
- Automatic focusing. *Q: What physical principles are used for it?*
- Image stabilization minimizing effect of a thrashing hand.
- 🔶 Built-in flash.
- Ability to capture a video sequence.
- Image resolution and its compression can be set.
- Image capturing in a RAW mode.
- The camera contains a processor dealing with basic image processin/analysis operations, e.g, human face detection which serves for automatic selection of image points used for automatic focusing.

The job of a (photographic) lens



- The optical system (photographic lens) focuses the incoming energy (photons) and creates the image on the image sensor.
- The optical lens is a single, optically transparent device shaped to allow the light transmission and refraction while creating a desired optical outcome.
- The measured physical entity is the irradiance [W m⁻²] (informally brightness or gray value from a human perception point of view).
- The lens should mimic the ideal perspective transformation as much as possible (also projective transformation, pin-hole model).
- We will constrain to geometric optics in this simplified optics explanation. We leave wave and quantum optics models aside.

Approximation by geometric optics



It is one of several possible approximations.

Assumptions:

- The involved wavelengths of the electromagnetic irradiation (here a frequency sub-band of it = light) are very small as compared to sizes of used optical and mechanical elements.
- The energy of photons (from the quantum theory point of view) are small with respect to energetic sensitivity of involved sensors.

Geometric optics is a rough approximation. Geometric optics is important for daily life technology. It is also interesting from the point of view of the historic development of opinions in physics.

Recommended reading: Feynman R.P, Leighton R.B., Sands M.: Feynman Lectures on Physics, 3 volumes, (1963-1965).

A pin-hole camera



- 15th century, the architect Filippo Brunelleschi from Italian Florence (1377-1446), a tool for drawing perspective images.
- 16th century, pin-hole camera, in Latin camera obscura.
- 1822 The Frenchman J.-N. Niepce added a photographic plate to the pinhole camera \Rightarrow the first photograph was born.



Convention: We consider left to right direction of light.

The size of a hole in a pin-hole camera

The interplay of contradictory phenomena.

- a. The bigger hole passes more light but blurs the image.
- b. The small hole causes diffraction and the image will be blurred too.
- c. The optimum exists, in which the image is least blurred. Example: For f=100 [mm] and $\lambda=500$ [nm], the optimal diameter of the hole is 0.32 [mm].

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Why are lenses used?



The pin-hole camera:

- Collects only a few photons (light).
- Troubles due to diffraction due to the hole.
- No abberations (almost).



The lens:

- Collects more photons (light).
- + Have to be in focus.
- Suffers from abberations.



The (optical) lens from a physics point of view

 Behavior of a lens was explained by the Dutch mathematician Willebrord van Roijen Snell (1580–1626), who formulated the light refraction law on the boundary of separation of two contacting substances in the year 1621.

$$n = \frac{n_1}{n_2} = \frac{\sin \alpha_2}{\sin \alpha_1}$$
, where *n* is the refractive index.

• n for a yellow light λ =589 [nm] on the boundary between the vacuum and X:

X = air 1,0002; water 1.333; crown glass (a small diffusion of light, a small refraction index) 1.517; lead optical glass 1.655; diamond 2.417.

 There is an elegant derivation of the Snell's refraction law, which uses (approximate) Fermatt's principle of the shortest time from the year 1650, see Feynman's Lectures on Physics.





Lens



- A lens forms an image by focusing light from the scene.
- Different rays of an incident light beam are refracted through different angles, so that an entire beam of parallel rays can be caused to converge on, or to appear to diverge from, a single point.



The lens, mathematical model



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The thin lens equation in a Newtonian form $\frac{1}{f} = \frac{1}{z'+f} + \frac{1}{f+z}$ or rewritten to a simpler expression $f^2 = z z'$







Ray diagrams

Ray diagrams simplify lens operation by considering three distinct cases:



- A. A ray passing through the optical centre of the lens;
- B. A ray parallel to the principal axis, which refracts through the lens, passing through the principal focus;
- C. A ray passing through the principal focus (on the same side as the object) and being refracted through the lens, emerging parallel to the principal axis.

It is useful to distinguish between two categories of images (also in ray diagrams):

- **Real images** are produced from actual rays of light coming to a focus (e.g. a film projected onto a screen);
- Virtual images are produced from where rays of light appear to be coming from (e.g. a magnifying glass image).

We will show diagrams representing image formation from an object positioned. Let F be the focal point on the object side (left).

- \blacklozenge object positioned between F and the lens
- ullet object positioned at F
- lacksim object positioned between F and 2F
- ullet object positioned at 2F from the lens

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Arrangements, positioning of the object

Magnifying glass. Object between F and the lens; virtual, upright, magnified



 Object at F; parallel rays, image at infinity = no image



• **Projector**. Object between F and 2F.



• 1:1 Copy machine. Object at 2F.

Similar image to the image above. This time the image is the same distance behind the lens as the object is in front. As before, the image is inverted and real, but is the same size as the object.

• **Camera lens**. Distance to the object > 2F, usually $\gg F$.

Normal lens, wide-angle lens, telephoto lens.



Arrangements, telescope, microscope

Refracting astronomical telescope

- Used to see large and distant objects. Large objective aperture.
- Focal length of the eye piece lens is smaller than focal length of the objective.
- Objective focal length of the objective is large for higher magnification.



Compound microscope

- Used to see close, very small objects. Small objective aperture.
- Focal length of the eye piece lens is grater than focal length of the objective.

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 Objective focal length of the objective is large for higher magnification.



Thick (composed) lens the approximation of the optical system

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The compound lens is used to suppress optical abberations.

Principal optical abberations: vignetting (natural, optical, mechanical), chromatic, spherical abberations, coma, astigmatism, and geometric (radial/tangencial) distortions. We will treat optical abberations later in this lecture.

(Object space) telecentric lens



- Only chief rays used, i.e. oblique rays passing through the center of the aperture stop placed at focal point. These rays are \approx parallel to the optical axis.
- The input lens has to have bigger diameter than measured object.
- Useful when measured object changes its position or the object is 'thick' because magnification is independent to the distance from the lens.





Diaphragm (also aperture stop)

- A thin planar opaque object perpendicular to the optical axis with an aperture at its center.
- The diaphragm role is to stop part of the light energy (rays) from reaching the focal plane.
- The aperture size is adjustable either manually or it is motorized with commercial lenses in cameras and video cameras.
- The adjustable aperture is often called the iris diaphragm, analogically to the iris in human eye. The adjustable diaphragm often consists of adjustable blades.



One possible construction of the diaphragm.

Diaphragm 2, examples







f2.8

f8



Depth of focus



- Depth of focus explains, why is it possible to shift the image plane to the right a little in the direction of optical axis in the image space and still have the image in focus. It is because of a finite size of one pixel on a sensor or a definite size of the grain in the film.
- The dispersive circle has a diameter ε .



Depth of field



Depth of field determines the range of distances from the center of projection in the object space, in which the objects are shown in focus. This is the parameter, which is of practical interest for the photographer.



Aperture stop influence to the field of focus (1)







a small aperture, a large depth of field

a large aperture, a small depth of field



Aperture stop influence to the field of focus (2)



Lens Canon EF 50 mm f/1.8 II with maximum aperture 1:1.8.

Influence of the focal length to the depth of field









Parameters of lenses (1)

Focal length – fixed, adjustable (zoom) manually or motorized. **Diaphragm**

- Aperture is the maximum diameter of a light beam that can pass through an optical system. The size of aperture is controlled by the size of diaphragm.
- Diaphragm can be fixed, adjustable manually or motorized.

Lens connecting

- C the distance between the back of the lens and the chip is approx. 17 mm.
- CS approx 12 mm, the other parameters are the same.
- Lens for C mount can be adjusted to CS mount by an extension ring 5 mm thick. It does not work from CS to C.









Parameters of lenses (2)



- **Focusing** Fix focus (e.g., web cameras, mobile phones), manual or motorized focusing.
- **Distances in which object is in focus** can be changed by extension rings in the expense of deteriorated optical properties.
- Format which is the biggest chip usable; 1", 2/3", 1/2", 1/3", 1/4".
- **Thread for a filter** e.g., a clear filter is used to protect the lens.
- **Radial distortion** is not given in technical sheets but it is important for measurement applications. Lenses with short focal length have typically bigger radial distortions (several pixels).

Camera Sensor Formats



Courtesy Markus Kohlpaintner (figure)

Natural vignetting

• The term $\cos^4 \alpha$ describes a systematic optical abberation named natural vignetting.

The derivation of the related Irradiance equation can be found in the lecture about image capturing from the physics point of view.

- The natural vignetting phenomenon describes the situation, in which the rays refracted at higher angle α with respect to the optical axis.
- This error (natural vignetting) is more pronounced with wide angle lenses than with telephoto ones.
- Natural vignetting is a systematic error. It can be undone if the camera and the scene is radiometrically calibrated.







original



vignetting

Optical vignetting



- The compound lens thickness thickness ranges from several millimeters to several centimeters. This is the reason, that why not all rays can hit the lens aperture opening.
- The phenomenon is more pronounced for a more open aperture stops.



Mechanical vignetting

- Only inattentive users suffer from mechanical vignetting.
- The lens hood must match the particular lens..





Chromatic abberations

- Chromatic aberrations are caused by the dependence of the lens refraction index on the light wavelength.
 This property is desired in the prism for the light decomposition. However, it is undesirable for lenses.
- The lens brings different colors (wavelength) to a focus at different points on optical axis.
- Two types: Longitudial/axial (Fig. line 1) and lateral/traverse (Fig. line 2). The aberration is more pronounced at the margins of the image.

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Chromatic abberations, doublet lens correction

- The abberation is rectified while manufacturing the lens.
- A pair of doublets is used as a building element, i.e. the lens is composed of two pieces from two different materials, the crown and the lead (flint) glasses, optical glasses with, respectively, relatively low and high refraction indices.



Doublet lens correcting for longitudinal chromatic abberation.



Chromatic lateral abberation, a practical view





Near to the optical axis Center of the image



Far from the optical axis Image rims

Chromatic abberation, extreme illustration







Bad quality lens of a peephole in the US motel. Projected sunset on the opposite side of a dark roon.
Astigmatism



- Astigmatism is an optical aberration that occurs when rays lying in two perpendicular planes to the optical axis (sagittal = horizontal; tangential = vertical) have different foci.
- This causes blur in one direction that is absent in the other direction. If we focus the sensor for the sagittal plane, we see circles become ellipses in the tangential direction and vice versa.



Coma



- Coma (from Latin comet) applies to rays entering the lens at an slanted angle. These rays do not quite converge at the focal plane.
- Coma can be demonstrated by tilting a single lens under the sunlight. If the lens optical axis directs to sun, the projected sun is circular. If the optical axis is tilted the projected sun has elongated shape, like a comet.
- Coma is positive when off-axis rays focus furthest from the axis, and negative when they are closest.



Spherical aberration

Spherical lenses are very common because they are relatively easy to manufacture.
 The spherical shape is not ideal for perfect imaging. Collimated rays entering the lens at different distances from the optical axis will converge to different points at optical axis, causing an overall loss of focus.

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- The blur effect increases towards the edge of the lens.
- To reduce the problem, aspherical lenses are often used.



 $Courtesy \ http://www.opto-e.com/$

Petzvald field curvature



- Field curvature aberration describes the fact that parallel rays reaching the lens from different directions do not focus on a plane, but rather on a curved (Petzval) surface (rotational paraboloid).
- This causes radial defocusing, i.e. for a given sensor position, only a circular crown will be in focus. The aberration manifests itself as geometric radial distortion described next.



Radial (geometric) distortion

- It is the prevalent distortion. It is pronounced more with wide-angle lenses.
- (x', y') are uncorrected point coordinates measured in the image; (x, y) are corrected coordinates; (x_0, y_0) are coordinates of the principal point; (Δ_x, Δ_y) are elements of the correction, and r is the radius, $r = \sqrt{(x' - x_0)^2 + (y' - y_0)^2}$.
- The distortion is often approximated by a polynomial of the even order (why?), often only 2nd order.

$$\Delta_x = (x' - x_0) \left(\kappa_1 r^2 + \kappa_2 r^4 + \kappa_3 r^6 \right), \Delta_y = (y' - y_0) \left(\kappa_1 r^2 + \kappa_2 r^4 + \kappa_3 r^6 \right).$$





pincushion

Radial distortion, practical illustration









barrel

without distortion

pincushion

Radiometric image formation in a camera



Courtesy: Sergey Alexandrov

Low brightness dynamic range illustration



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Luminance $[cd/m^2]$: night with Moon light 10^{-2} ; indoor lighting 10^2 ; day light (overcast/clear sky) $10^3/10^5$.

Courtesy: Sergey Alexandrov



- Incoming radiation (photons) in converted in the semiconductor mass into charge couples, electron-hole.
- The semiconductor is in a static electric field. The Electron-hole couples are converted into a short current impulse.
- The current impulse must be amplified and processed. E.g., in a CCD element the impulse is used to charge a capacitor.

Photodiode and MOS structure



Cross cut of two main principles for current generation and storing the charge.



CCD architectures





CCD chip, properties of the technology



- + Linearity: CCD sensors explore conversion of a photon to the couple electron-hole. The obtained charge is integrated in a capacitor.
- + Low noise: is given by the integral character of the measurement. Uncooled chip with TV read-out has SNR approx. 60 dB.
- + Efficiency: Current sensors have hight energetic efficiency approx. 40%, i.e. every third photon generates one couple electron-hole.
- **Read-out:** only from the whole chip at once.
- Limited range of intensities: is given by the maximal capacity of individual capacitors...

CMOS chip, properties of the technology



http://www.ims-chips.de/products/vision/hdrc_alt/hdrc_ima.html
http://www.imec.be/bo
http://www.vector-international.be/C-Cam/cmosccd.html

- + Logarithmic sensitivity: CMOS sensors are based on the photo diode principle. They measure a current in a read-out instance.
- + **Read-out:** possible in arbitrary order, e.g. only the region of interest can be read-out.
- + Camera and processor on the same chip: CMOS technology is well mastered (processors, memory). Smart cameras.
- Higher noise:

Cameras, user's view (1)



- Spatial resolution: number of pixels in a row and in a column. TV CCIR/PAL 768×576. TV RS170/NTSC 640×484. Non-television cameras also 2000×2000, keep increasing.
- Resolution in intensity: given in bits for digital cameras, output typically 8 bits also 12 bits. For analog cameras SNR, usually >50 dB.
- **Sensitivity:** v lux. Should be recalculated according to used diaphragm and AGC.
- **AGC:** Automatic Gain Control; yes/no, can be switched off?, manual control of gain.
- Shutter: commonly from 1/50 s to 1/10000 s.

Cameras, user's view (2)



- Format: size of the photosensitive chip. Given either in inches of the equivalent vidicon tube diameter or in mm. 1/2" corresponds to 4.8×6.5 mm.
- Shape of a pixel: square pixel vs. non-square pixel.
- Output for automatic diaphragm:
- **AWB:** Automatic White Balance. Changes ratio of R and B with respect to G.
- Gama correction: fixed/adjustable. Direct signal γ = 1. Typically γ = 0, 45 (enhances black). Compensates intensity conversion function of the CRT (Cathode Ray Tube) and adjusts it to the sensitivity of a human eye.
- Lens thread: C mount / CS mount.

Interlaced/non-interlaced scanning





Interlaced.

Non-interlaced.

Signal, interlaced/non-interlaced scanning



Interlaced.

Non-interlaced.

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Electronic shutter



Shortened exposition is used either if there is too much light or if fast events have to be captured.



Flash light and suppression of ambient light





 The instant of the flash is set when the shutter is open.

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- The ration between the integral of ambient intensity during the shutter opening and integral of the flash intensity gives the influence of ambient light.
- + LED are often used as cheap 'flash light'.

Color cameras setups



- Manual change of color filters in front of the monochromatic camera lens.
- Three chip cameras an incoming light is divided to a appropriate chip using color filters and semitransparent mirrors.
- One chip camera has filters directly on a chip. Spatial resolution in color resolution is smaller than coresponds to the number of pixels.

Arrangement of color filters in single chip cameras



R	G	R	G	
G	В	G	В	
R	G	R	G	
G	В	G	В	

С	Y	С	Y	
М	G	М	G	
С	Y	С	Y	
М	G	М	G	

Additive color model.

Subtractive color model.

Color scanner















b



С

a









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virtual image at infinity
































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C-Mount







Camera Sensor Formats















- RGB color rays
 Optical axis
- ---- Best focus point
































pincushion

barrel















Camera response function





10 ms

Auto EV





















R	G	R	G	
G	В	G	В	
R	G	R	G	
G	В	G	В	

С	Y	С	Y	
Μ	G	Μ	G	
С	Y	С	Y	
Μ	G	Μ	G	

