# Image restoration with known degradation

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## Outline of the lecture:

- Linear model of the image degradation.
- Three useful degradation models..
- Inverse filtration.
- Pseudoinverse filtration.
- Wiener filtration.
- Examples.

# Image restoration ideas

- Image restoration is a collection of techniques aiming at using a priori knowledge about the image degradation mathematical model.
- It is attempted to find a model of image degradation and its parameters for a specific class of images provided by an application.
- Image restoration induces the solution of the inverse task to image degradation modeling.
- The linear model of image degradation is used (a convolution across the entire image).
- There are two categories of involved methods: deterministic and stochastic ones.

# The model of the degradation – the convolution



$$g(x,y) = \int\int_{(a,b)\in\mathcal{O}} f(a,b)\, h(a,b,x,y)\, \mathrm{d}a\, \mathrm{d}b + \nu(x,y)\ ,$$

where $f(x,y)$ is the undegraded image (and also unobservable),

$g(x,y)$ is the degraded image,

$\nu(x,y)$ is additive noise, and

$h(x,y)$ is a shift-invariant convolution model of image degradation.

$$\begin{aligned} g(i,j) &= (f * h)\,(i,j) + \nu(i,j)\ . \\ G(u,v) &= F(u,v)\, H(u,v) + N(u,v)\ . \end{aligned}$$

# Three degradations which can be modelled well

1. Out-of-focus lens.
2. Relative camera motion degradation (appears for longer opened shutter).
3. Atmospheric turbulence when the image is captured through a thick layer of air, e.g. in remote sensing or in astronomy.

Individual degradations will be expressed as the convolution kernel $H(u,v)$ in the formula

$$G(u,v) = F(u,v)\,H(u,v) + N(u,v)\,.$$

# Relative motion between the object and the camera



- We express the simplest special case for illustration.

  We consider a constant speed $V$ of the moving object in direction of the axis $x$ with respect to the camera which happened while the shutter was opened for the time $T$.

- The model of the degradation is

$$H(u,v) = \frac{\sin(\pi V\ T u)}{\pi V\ u}\,.$$

# Out-of-focus lens

- The blur caused by the out-of-focus thin lens for small depth of focus is expressed as
$$H(u,v) = \frac{J_1(a\,r)}{a\,r}\,,$$
where $J_1$ je Bessel function of the first order,
$r^2 = u^2 + v^2$,
$a$ is a shift in the image.
- The last parameter $a$ shows that the model is not space invariant.

# Atmospheric turbulence

- The distortions are caused by heat inhomogeneities in the atmosphere (air shimmer) causing a slight light rays diffraction.
- The mathematical model was determined experimentally

$$H(u,v) = e^{-c(u^2+v^2)^{\frac{5}{6}}},$$

where $c$ is a constant given by the type of the turbulence.

- The constant $c$ is often determined experimentally for the particular class of applications.

# Inverse filtration (1)

$$G(u,v) = F(u,v)\,H(u,v) + N(u,v)\,.$$
$$F(u,v) = G(u,v)\,H^{-1}(u,v) - N(u,v)\,H^{-1}(u,v)\,.$$

- Works well for images which are not contaminated by noise.
- If noise is not negligible, the additive error appears in the formula pronounced for frequencies, in which the the inverse filter has a small amplitude (analogy to dividing by zero).
- This effect appears usually for higher frequencies. Consequently, the image restored by the inverse filter has often blurred originally sharp edges.

# Inverse filtration (2)

original

blurred

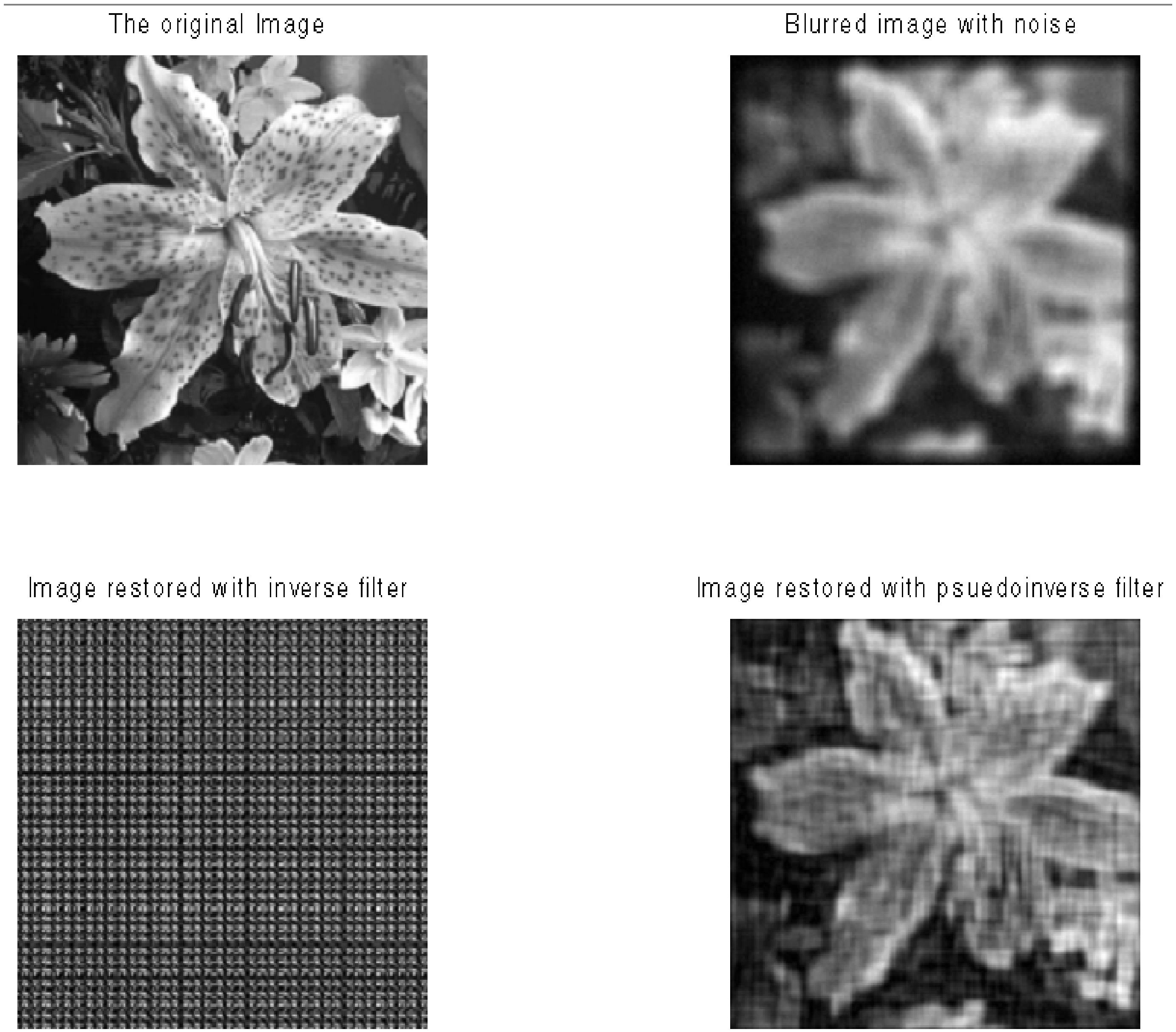
inverse filtrated

# Inverse filtration (3)

$$F(u,v) = G(u,v)\ H^{-1}(u,v) - N(u,v)\ H^{-1}(u,v)\ .$$

- Changes in the noise amplitude in the image influence the resulting image negatively.
- The module of the complex function $H(u,v)$ decreases with growing frequencies faster than $N(u,v)$. As a result, the noise-caused artifacts may overweight the useful information in the image.
- The way around is often to use the inverse filtration in a neighborhood of the origin of the $u,v$ plane, where $H(u,v)$ dominates safely. The result is often useful.

# Pseudoinverse filtration



# Wiener filtration (1)

- Works for non-negligible noise, the stochastic properties of which can be estimated. It is assumed that the noise is independent on the signal and that the stochastic process is stationary in the wider sense.
- Let $f$ be a undegraded (unobservable) image, $g$ be the observable degraded image and $\hat{f}$ be the estimate of the undegraded image.
- The problem is expressed as the optimization task. The undegraded image is the solution of the overdetermined system of linear equation which minimize the mean square error

$$e^2 = \mathcal{E}\left\{ \left( f(i,j) - \hat{f}(i,j) \right)^2 \right\} ,$$

where $\mathcal{E}$ denotes the mean value operator.

# Wiener filtration (2)



- Ideally, if no additional constrains are imposed to the solution then the estimate $\hat{f}$ is given by the conditional mean of the ideal image $f$ under the condition of the observed image $g$.
- The trouble is that the conditional probability of the ideal image $f$ under the condition that the observed image $g$ is available.
- In addition, the optimal estimate $\hat{f}$ is non-linearly dependent on the observed image $g$.

# Wiener filtration (3)

- The filter $H_W$ is sought, $\hat{F}(u,v) = H_W(u,v)\, G(u,v)$.
- The principle of orthogonality is used

$$\mathcal{E}\left\{[f(x,y) - g(x,y)]\, \nu(x',y')\right\} = 0\,.$$

- The above formula is expressed using correlation functions $R$

$$R_{\upsilon\nu}(k,l) = f(k,l) * R_{\nu\nu}(k,l)\,.$$

# Wiener filtration (4)

The expression in Fourier domain are used in order to explore power spectra instead of correlation functions.

$$H_W(u,v) = \frac{S_{f\nu}(u,v)}{S_{\nu\nu}(u,v)} = \frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_{\nu\nu}(u,v)}{S_{ff}(u,v)}},$$

# Example, motion blur


Left: The image degreded by the motion by 5 pixels in the $x$-axis direction.

Right: Outcome of the Wiener filter restoration.

# Example, out-of-focus lens


Left: Image captured by the out-of-focus lens.

Right: Outcome of the Wiener filter restoration.