

Digital image (for photography), basic concepts

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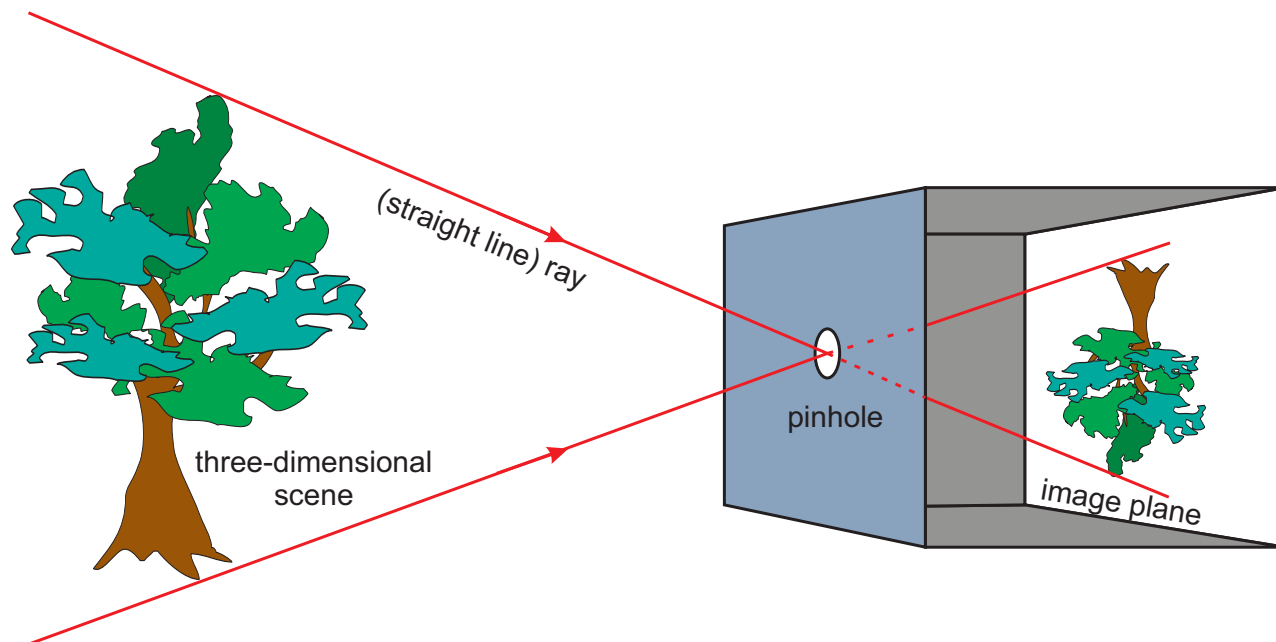
<http://people.ciirc.cvut.cz/hlavac>, vaclav.hlavac@cvut.cz

Outline of the lecture:

- ◆ Image, perspective imaging.
- ◆ Image function $f(x, y)$.
- ◆ Image digitization: sampling + quantization.
- ◆ Distance in the image, neighborhood.
- ◆ Contiguity relation, region, convex region.
- ◆ The entire image properties: brightness histogram, brightness, contrast, sharpness.

Image

- ◆ The **image** is understood intuitively as the visual response on the retina or light sensitive chip in a camera, TV camera, ...
- ◆ The image is often formed by a **perspective projection** corresponding to the intuitive pinhole model.



Convention:
We consider left to right
direction of light.

Perspective projection started in Italian Renaissance painting



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Filippo Brunelleschi created a perspective drawing to show his customers how the Church of Santo Spirito in Florence would look like.



Drawing in \approx 1420



View into the church built in 1434-82

Courtesy: pictures Khan Academy

Example of the perspective drawing tool from the 16th century



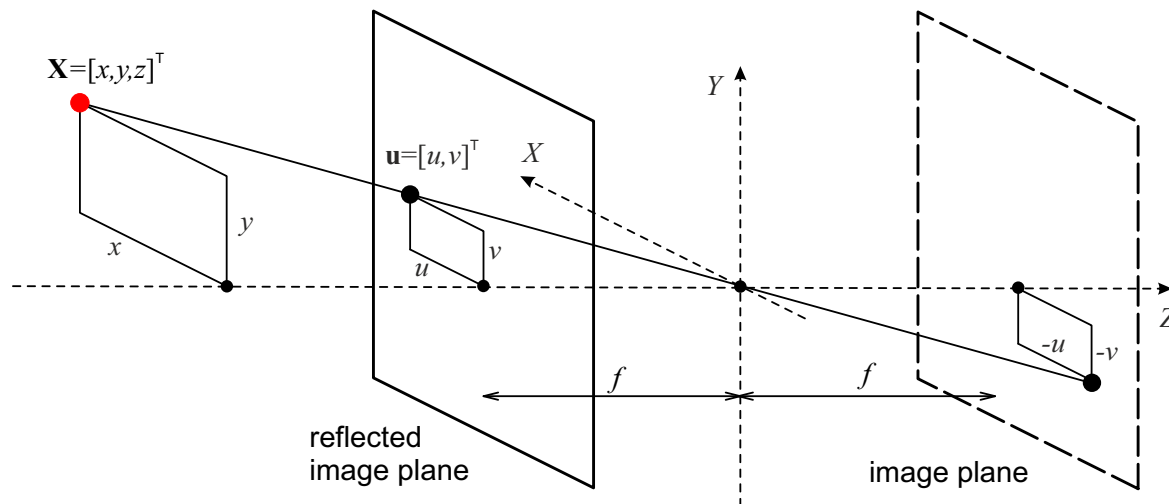
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Albrecht Dürer: Recumbent woman, wood engraving, 1525

Image function

- ◆ The image function is abstracted mathematically as $f(x, y)$, $f(x, y, t)$. It is the result of the perspective projection encompassing geometric aspects.



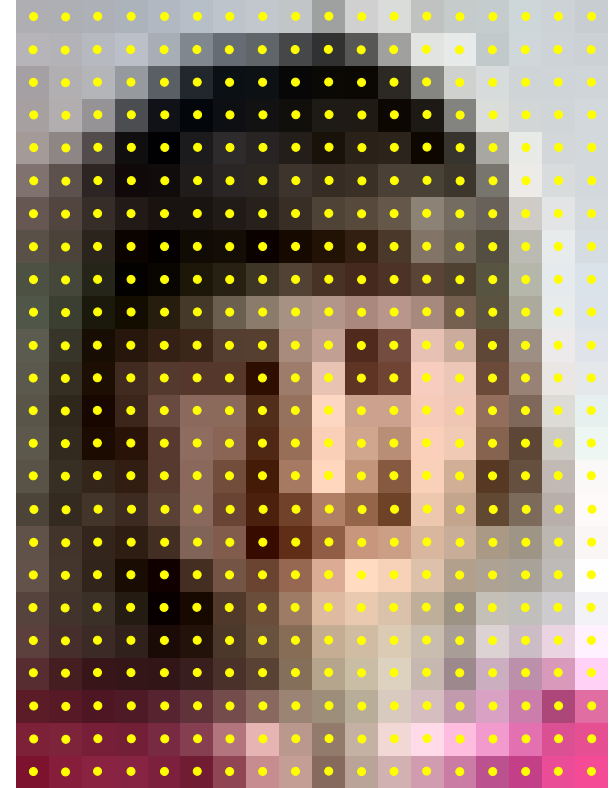
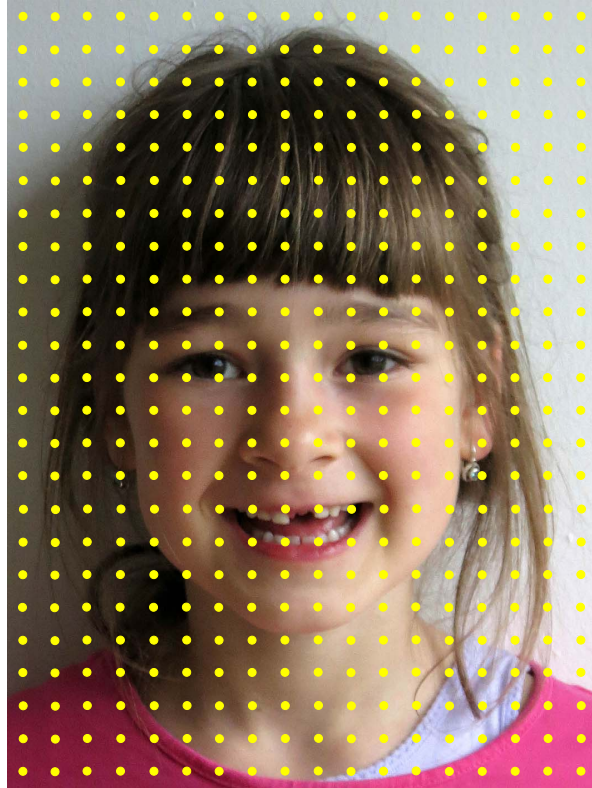
- ◆ Considering similar triangles: $u = \frac{x f}{z}$, $v = \frac{y f}{z}$. Instead of our derived 2D image function $f(u, v)$, it is usually denoted $f(x, y)$.
- ◆ The value of the image function matches color/intensity of a 3D point (a red dot in the figure above) in the scene, which is projected.



Continuous image and its mathematical representation

- ◆ (Continuous) **image** = the input (understood intuitively), e.g., on the retina or captured by a TV camera.
- ◆ Let us assume a gray level image for simplicity.
- ◆ The continuous **image function** $f(x, y)$. *Later, after digitization, a matrix of picture elements, pixels.*
- ◆ (x, y) are **spatial coordinates** of a pixel.
- ◆ $f(x, y, t)$ in the case of an **image sequence**, t corresponds to time.
- ◆ $f(x, y)$ is the **value of the image function** usually proportional to brightness, optical density with transparent original, distance to the observer, temperature in thermovision, etc.
- ◆ (Naturally) **2D images**: A thin specimen in the optical microscope, an image of a letter (character) on a piece of paper, a fingerprint, one slice in the tomograph, etc.

A pixel corresponds to a sample (not to a little square)



Digitization

- ◆ Digitization = sampling & quantization of the image function value (called also intensity).
 - Sampling selects samples from the continuous image function. The output is a finite number of are arranged in a discrete raster. The sample value is “continuous”, i.e. a real number.
 - Quantizing divides a real value of the sample into a finite number of values (also bins). It can be 256 gray-level values for gray-scale image.



Example: gray wedge



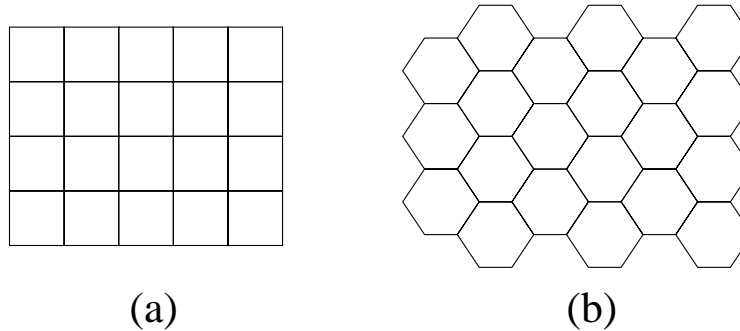
6 gray-level bins

-
- ◆ Digital image is often represented as a matrix.
 - ◆ Pixel = the acronym from the ‘picture element’.

Image sampling

Image sampling consists of two tasks:

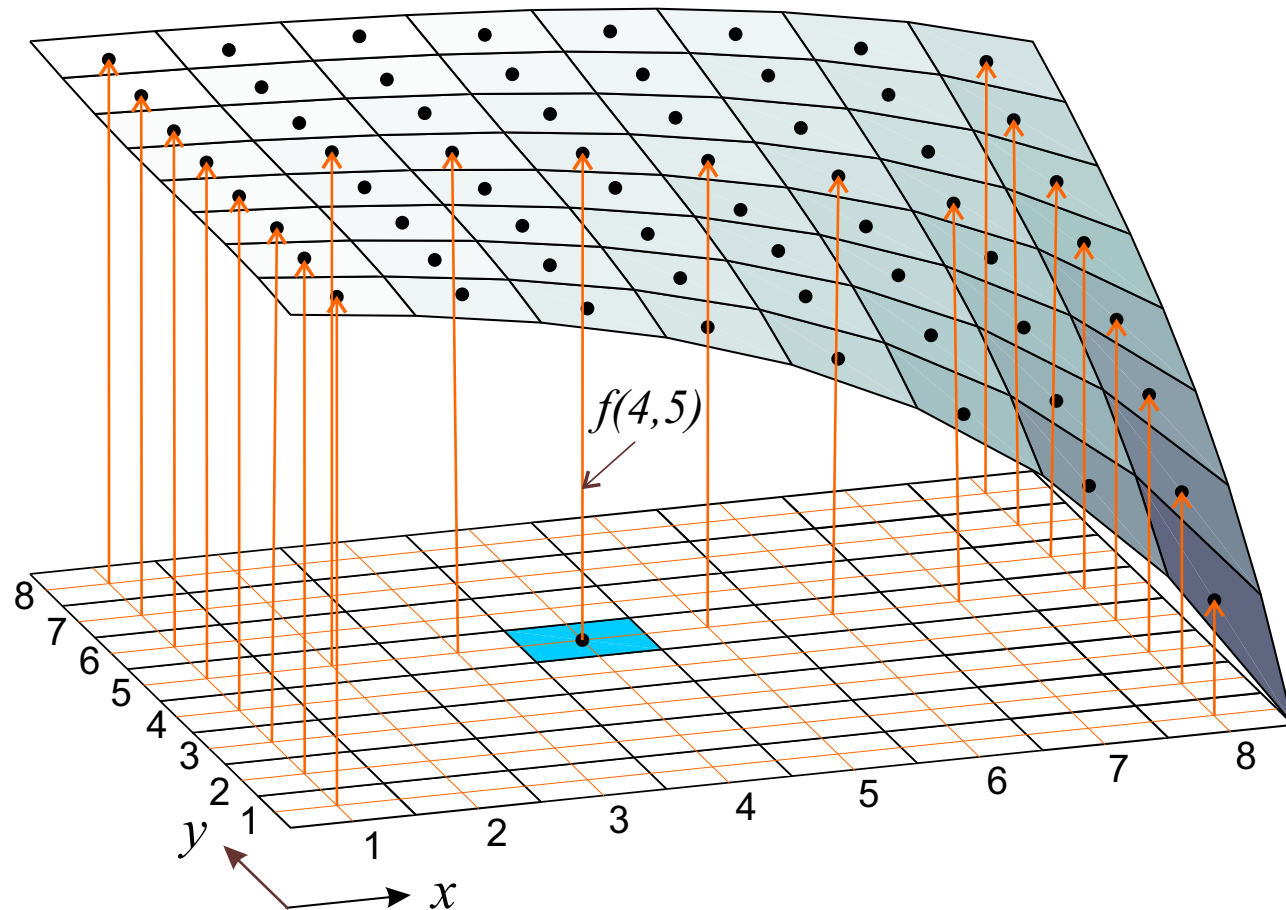
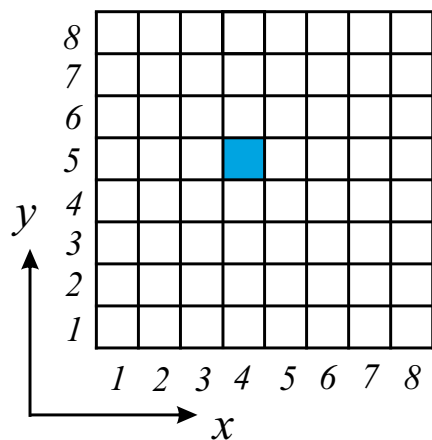
1. **Sampling pattern** (=arrangement of sampling points into a raster).



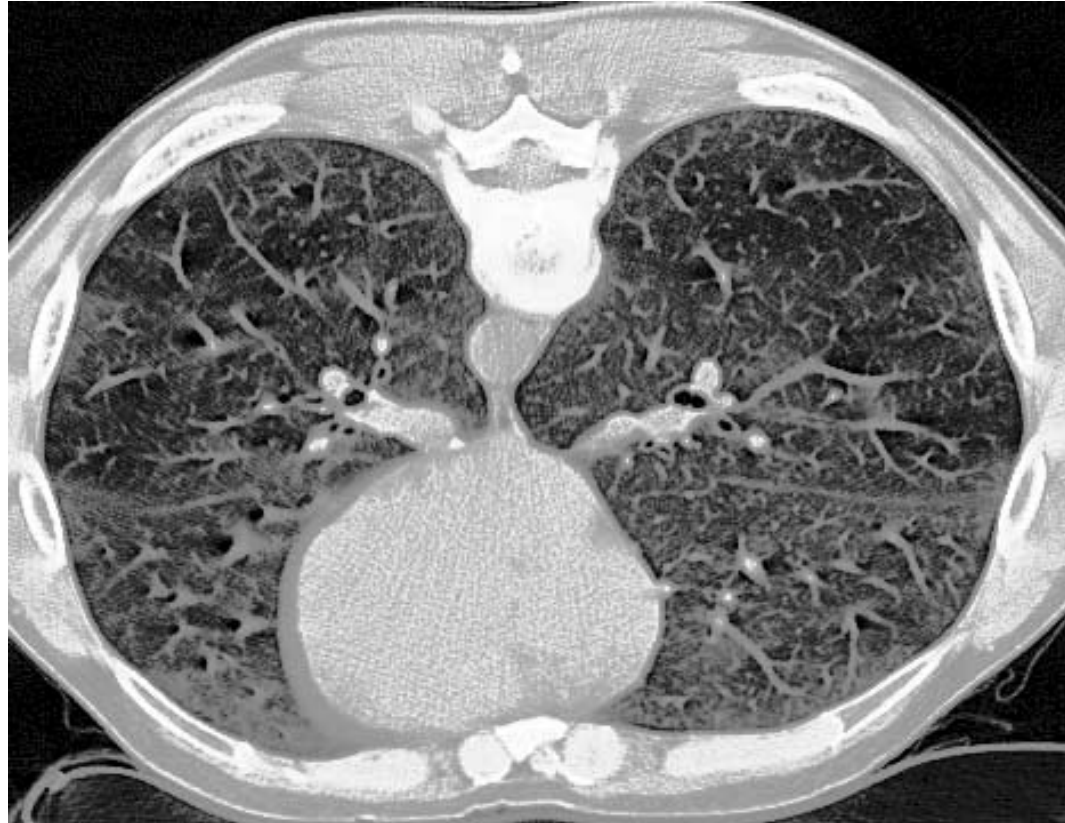
2. **Sampling rate/distance** (governed by Nyquist-Shannon sampling theorem).

- ◆ The sampling frequency must be $> 2\times$ higher than the maximal frequency of interest; in the sense: which would be possible to reconstruct from the sampled signal. We will be able to derive the theorem after we explain Fourier transformation.
- ◆ Informally: In images, the samples size (pixel size) has to be twice smaller than the smallest detail of interest.

Image sampling, illustration



Example of a digital image
a single slice from a X-ray tomograph



First image scanner, 1957



The SEAC Scanner
with control console in background



The first scanned image was the R. Kirsch's baby.

He used two thresholds and obtained three gray levels.

Russell Kirsch (*1929-†2020), SEAC and the start of image processing at the National Bureau of Standards. In: Annals of the history of computing, IEEE, vol. 20 (1998), p 7-13.

Image sampling, example 1



Original 256×256



128×128

Image sampling, example 2



Original 256×256

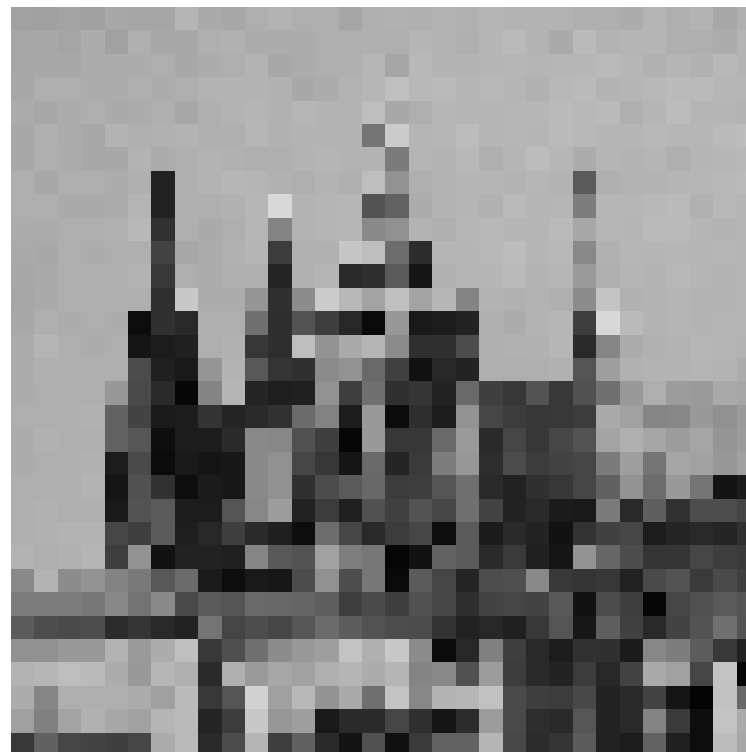


64×64

Image sampling, example 3



Original 256×256



32×32

Image quantization, example 1



Original 256 gray levels



64 gray levels

Image quantization, example 2



Original 256 gray levels



16 gray levels

Image quantization, example 3



Original 256 gray levels



4 gray levels

Image quantization, example 4 (binary image)



Original 256 gray levels



2 gray levels

The distance, mathematically

Function D is called the **distance**, if and only if

$$D(p, q) \geq 0, \quad \text{specially } D(p, p) = 0 \text{ (identity).}$$

$$D(p, q) = D(q, p), \quad \text{(symmetry).}$$

$$D(p, r) \leq D(p, q) + D(q, r), \quad \text{(triangular inequality).}$$

Several distance definitions in the square grid

Euclidean distance

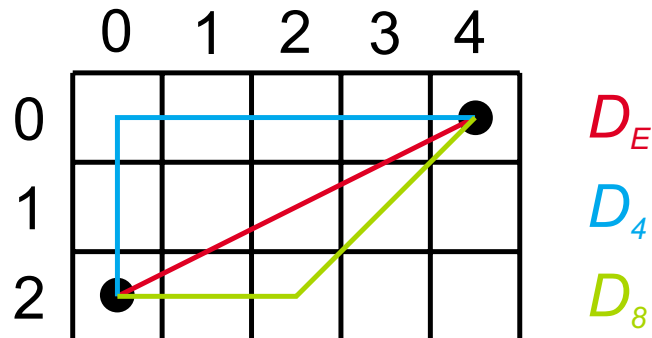
$$D_E((x, y), (h, k)) = \sqrt{(x - h)^2 + (y - k)^2} .$$

Manhattan distance (distance in a city with the rectangular street layout)

$$D_4((x, y), (h, k)) = |x - h| + |y - k| .$$

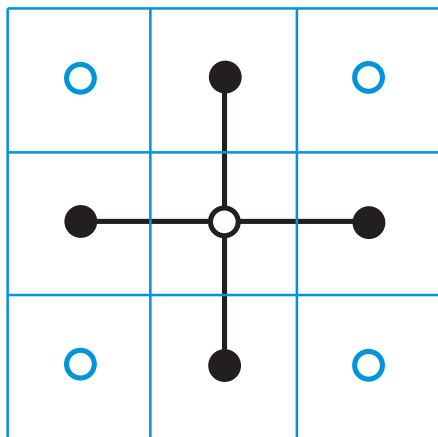
Chessboard distance (from the king point of view in chess)

$$D_8((x, y), (h, k)) = \max\{|x - h|, |y - k|\} .$$

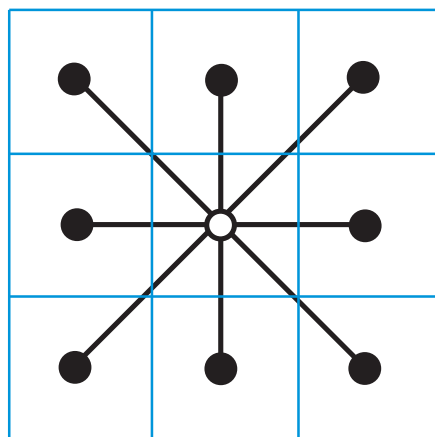


4-, 8-, and 6-neighborhoods

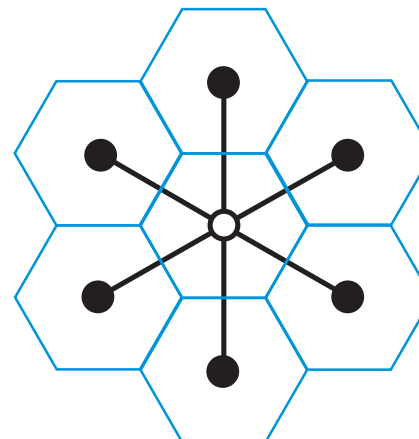
A set consisting of the pixel itself (shown in the middle as a hollow circle, called a representative pixel or a representative point) and its neighbors of distance 1 shown as filled in black circles.



4-neighbors



8-neighbors



6-neighbors

Binary image & the relation 'be contiguous'



black ~ objects

white ~ background

- ◆ *A note for curious. Japanese kanji symbol means 'near to here'.*
- ◆ Introduction of the concept 'object' allows to select those pixels on a grid which have some particular meaning (recall discussion about interpretation). Here, black pixels belong to the object – a character.
- ◆ Neighboring pixels are contiguous.
- ◆ Two pixels are contiguous if and only if there is a path consisting of contiguous pixels.

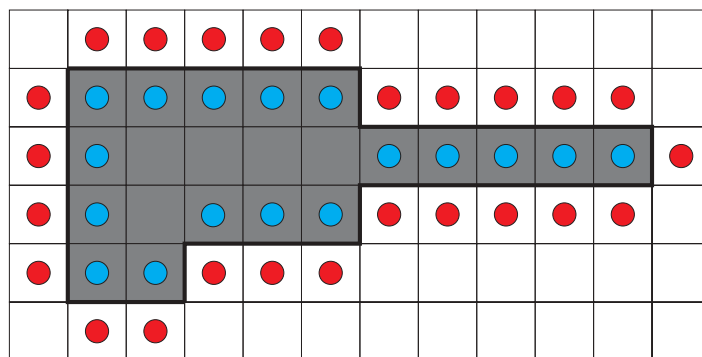
Region = contiguous set

- ◆ The relation ' x is contiguous to y ' is
 - reflexive, $x \sim x$,
 - symmetric $x \sim y \implies y \sim x$ and
 - transitive $(x \sim y) \ \& \ (y \sim z)$ $\implies x \sim z$. Thus it is an **equivalence relation**.
- ◆ Any equivalence relation decomposes a set into subsets called classes of equivalence. These are **regions** in our particular case of relation "to be contiguous".
- ◆ In the image below, different regions are labeled by different colors.



Region boundary

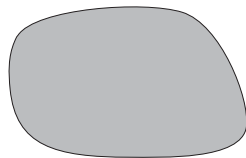
- ◆ The **Region boundary** (also border) R is the set of pixels is the set of pixels within the region that have one or more neighbors outside R .
- ◆ Theoretically, the the continuous image function \Rightarrow infinitesimally thin boundary.
- ◆ In a digital image, the boundary has always a finite width. Consequently, it is necessary to distinguish **inner** and **outer boundary**.



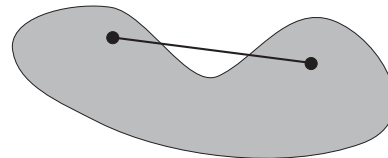
- ◆ Mind the terminology:
 Boundary (border) of a region \times edge, i.e., gradient of the image function \times edge element (edgel), i.e. a position with the significant magnitude of the gradient.

Convex set, convex hull

Convex set = any two points of it can be connected by a straight line which lies inside the set.



convex



non-convex

Convex hull, lake, bay.



Region



Convex
hull



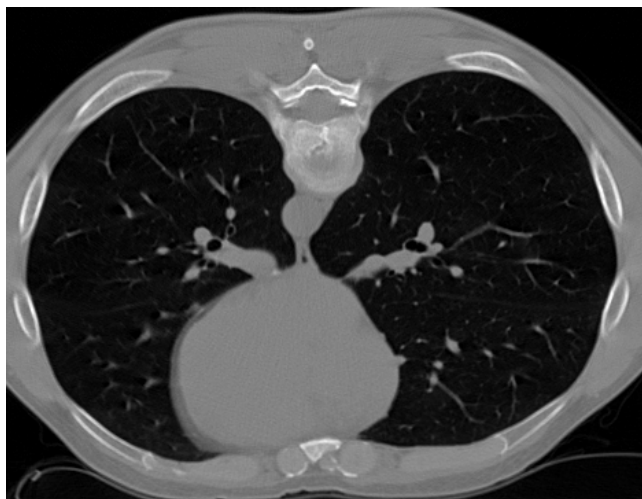
Lakes
Bays

Image properties used in its assessment/enhancing

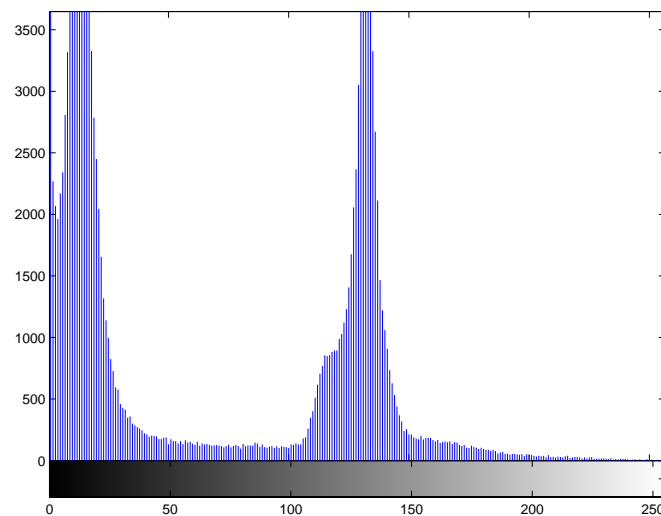
- ◆ When processing/enhancing an image by a human/machine, the image has to be assessed based on its properties.
- ◆ When a human observes the image, her/his image perception is influenced by a complex processing/interpretation in the brains and related **illusions**.
- ◆ We avoid such complexity pragmatically. The appropriateness of the image for human viewing is often simplified significantly to
 - one objective property – the **image histogram**, and
 - four subjective/objective properties: **brightness**, **contrast**, **color saturation**, and **sharpness**.

Image brightness histogram

- ◆ Let consider a gray scale image initially. We will use the human chest cross section in computer tomography image in the example below. Image histogram can be extended to color images too. Histograms are expressed independently in three color components, e.g. RGB.
- ◆ **Histogram** of brightness values serves as the **probability density estimate** of a phenomenon, that a pixel has a definite brightness.

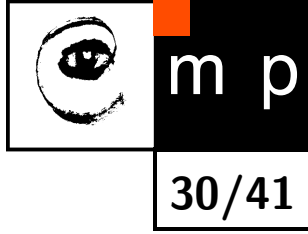


input image



its brightness histogram

Image properties from a human perception perspective



- ◆ When a human observes the image, her/his image perception is influenced by a complex processing/interpretation of the percepts in the brains and related **illusions**.
- ◆ We avoid such complexity pragmatically. The appropriateness of the image for human viewing is often simplified significantly to four properties of the image, i.e. the
 - image brightness,
 - image contrast,
 - image saturation (for color images only),
 - sharpness.

Brightness, contrast, color saturation, sharpness in the image



- ◆ **Image brightness** characterizes the overall lightness or darkness of the image.
- ◆ **Image contrast** characterizes the difference (separation) in luminance/color between objects or regions. For instance, a snowy fox on a snowy background has a low contrast. A dark dog on a snowy background has a high contrast.
- ◆ **Image color saturation** is a similar property to the contrast. However, instead of increasing the separation between object/regions in gray scale representation, the separation in the color domain is considered.
- ◆ **Image sharpness** is defined as the contrast of the edges, i.e. in the direction of the image brightness gradient. When we increase the sharpness, we increase the contrast only near the edges, proportional to the size of the gradient. In parts of the image that change little, we change the value of the image function less.

To support imagination, we use the analogy of (image) \leftrightarrow (landscape) or (brightness value) \leftrightarrow (height in the landscape at the relevant location). Then sharpening will increase the slope of the slopes, more for steep slopes and less for gentle slopes.

Towards enhancing a single image

- ◆ Let consider a **common practical task**. Let have only a single captured digital image. A human observer is not satisfied with its appearance manifested by image brightness, contrast, color saturation or sharpness.
- ◆ The **cause** could be that the **scene lighting** was not appropriate, the **image sensor dynamic range** was not high enough, the **objects were not distinguished** from the background, etc.
- ◆ A **human influences** namely the brightness, contrast, color saturation or sharpness when processing the already captured digital image, e.g. in PhotoShop.
- ◆ Let us illustrate the need of the image assessment/enhancing on **practical illustrative examples**.

Introducing the image used in experiments

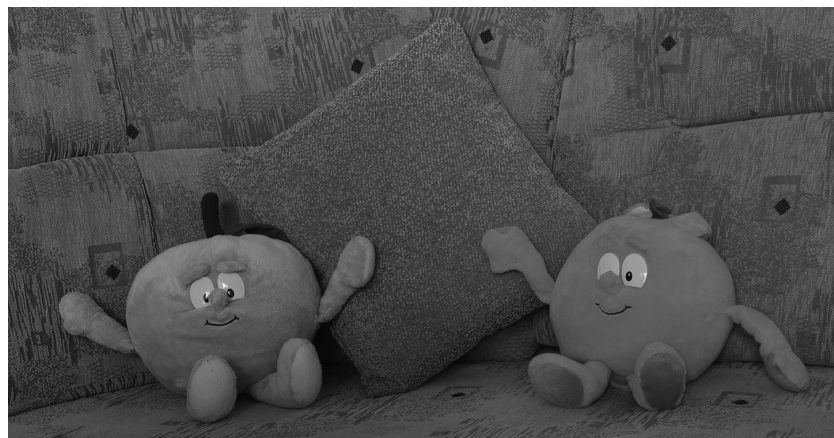
I captured the input color image of three objects on a light green sofa background. The scene shows deliberately one object (a square cushion) similar in color to the background and two objects, two plush toys, differing from the background in color. The first toy has different green color than the sofa and the second toy is orange.



We begin with gray level images, a conversion

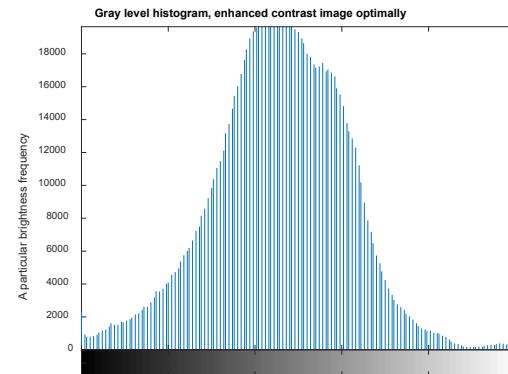
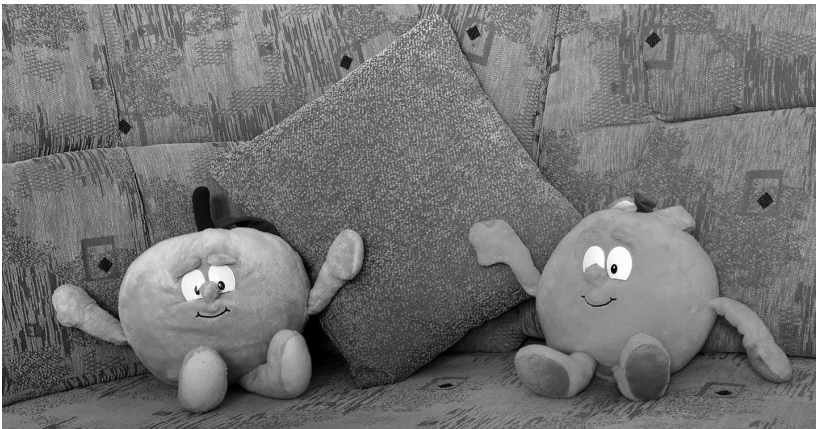
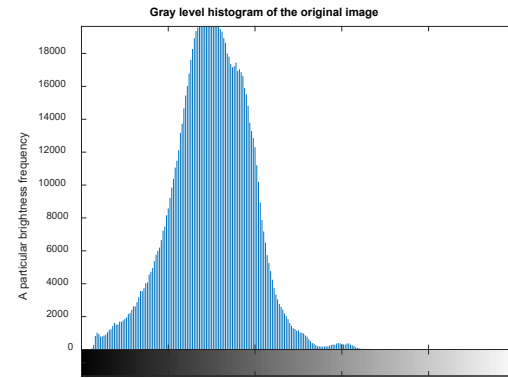
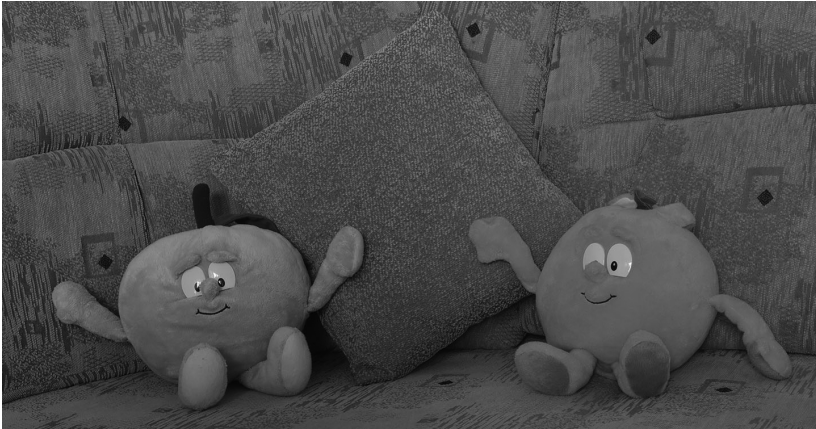


Original color image

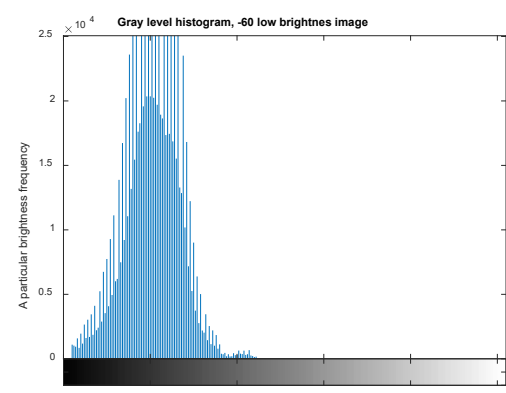
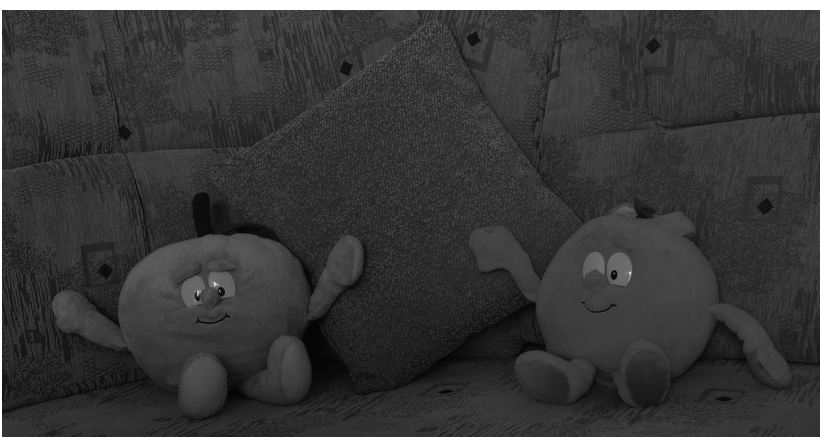
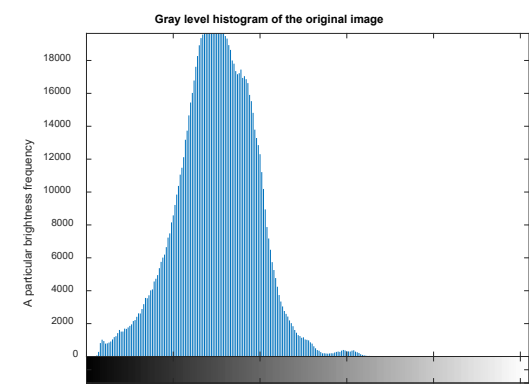
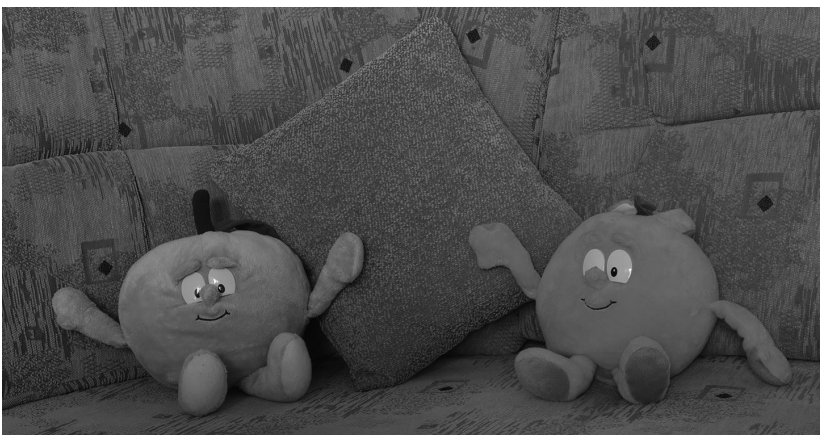


In the gray scale

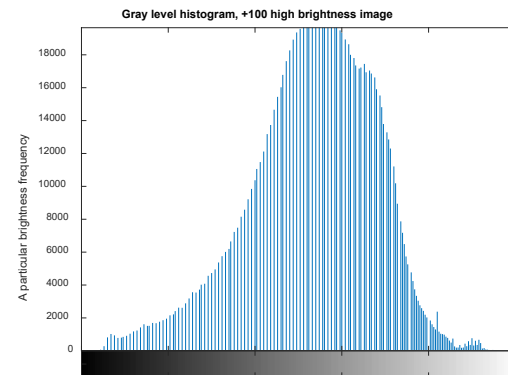
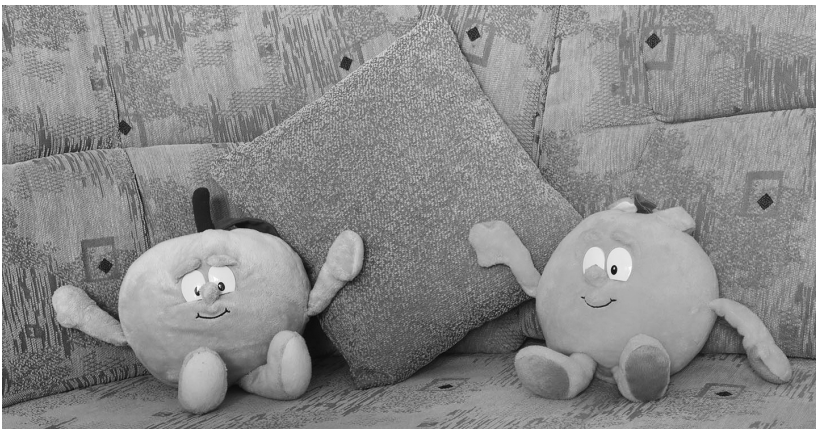
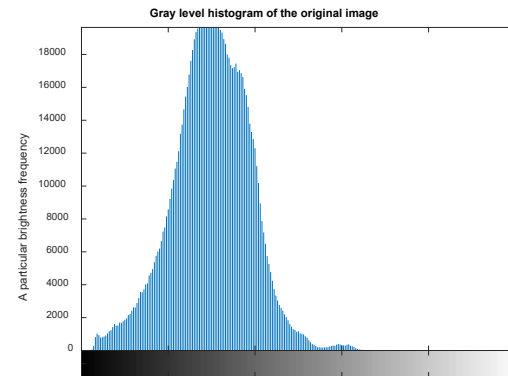
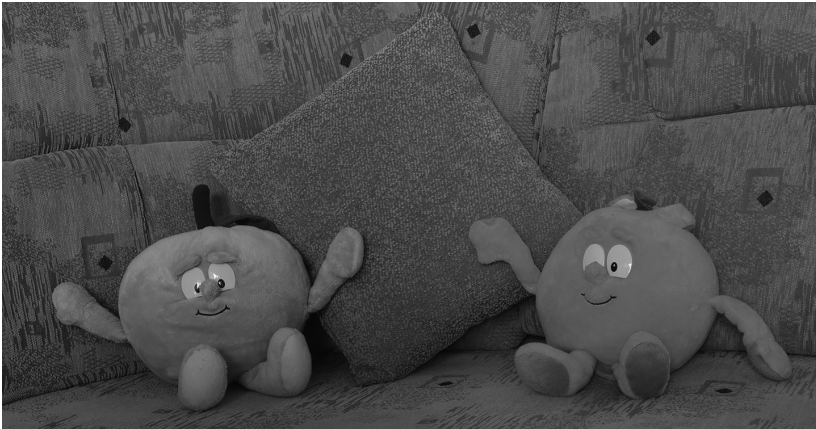
Optimally increased contrast illustrated



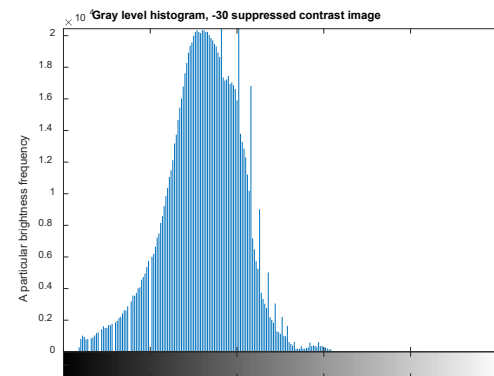
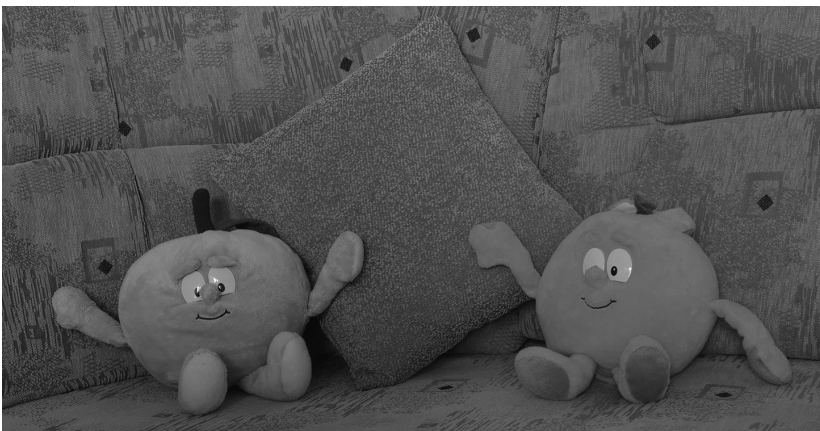
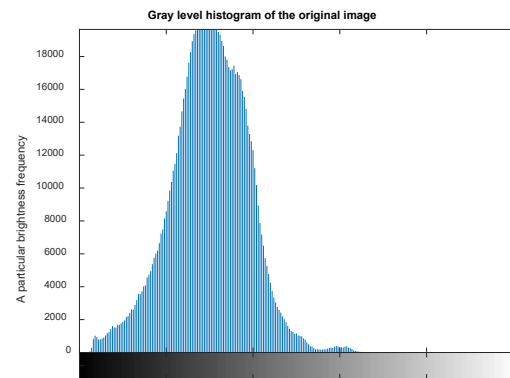
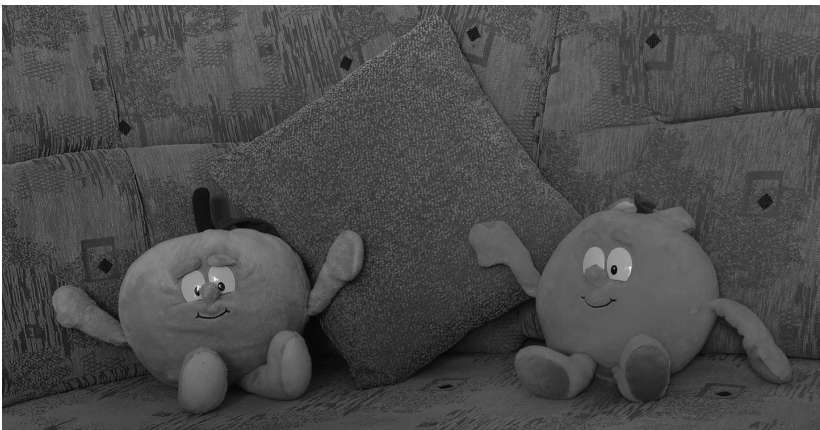
Low brightness illustrated



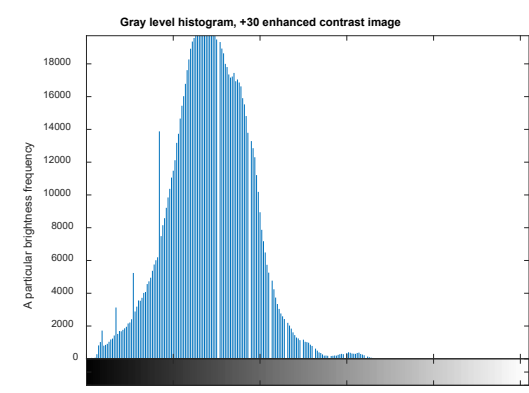
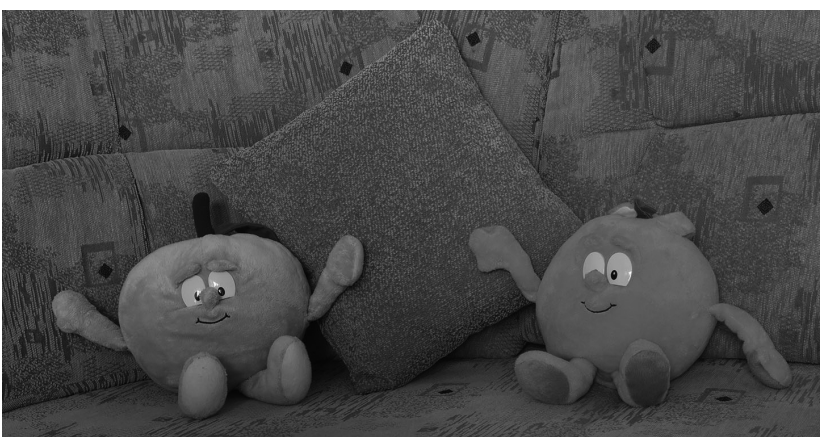
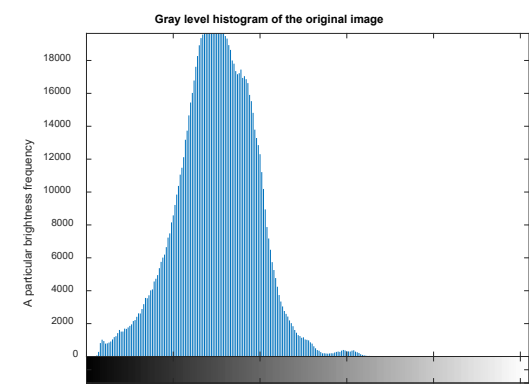
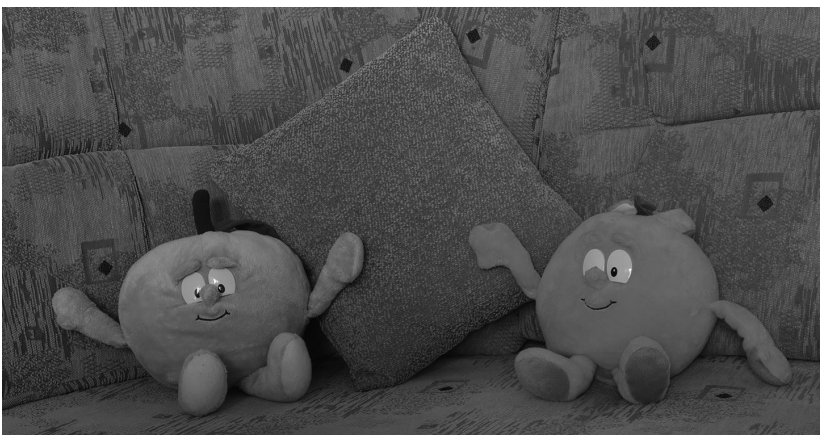
Increased brightness illustrated



Suppressed contrast illustrated



Enhanced contrast illustrated



Color saturation illustration

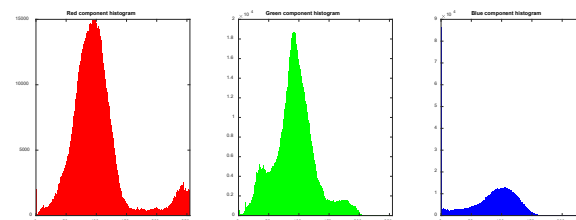
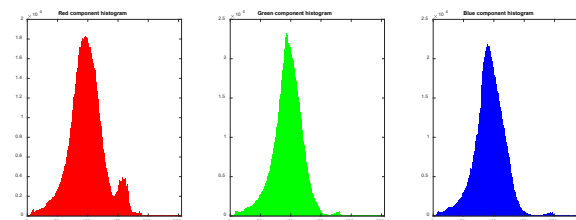
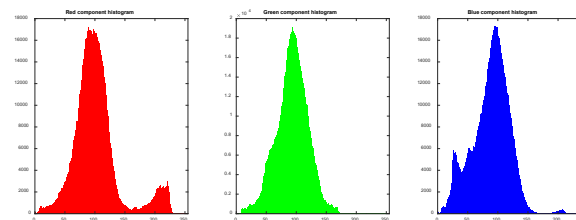
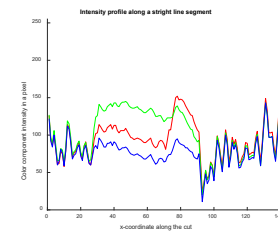
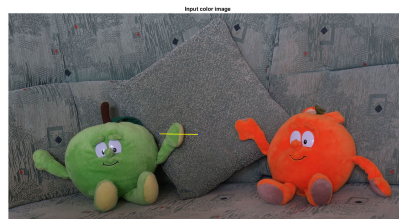
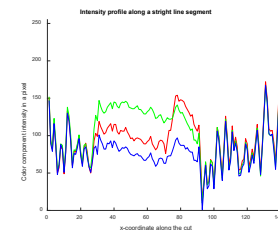
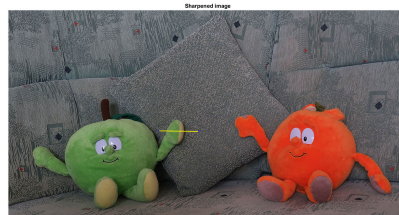


Image sharpness illustrated

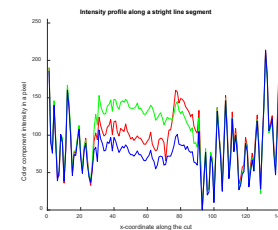
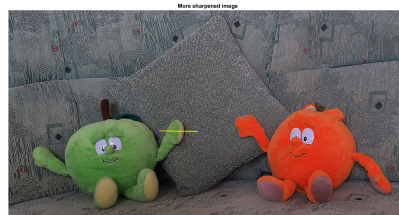
Input image



Moderate sharpening



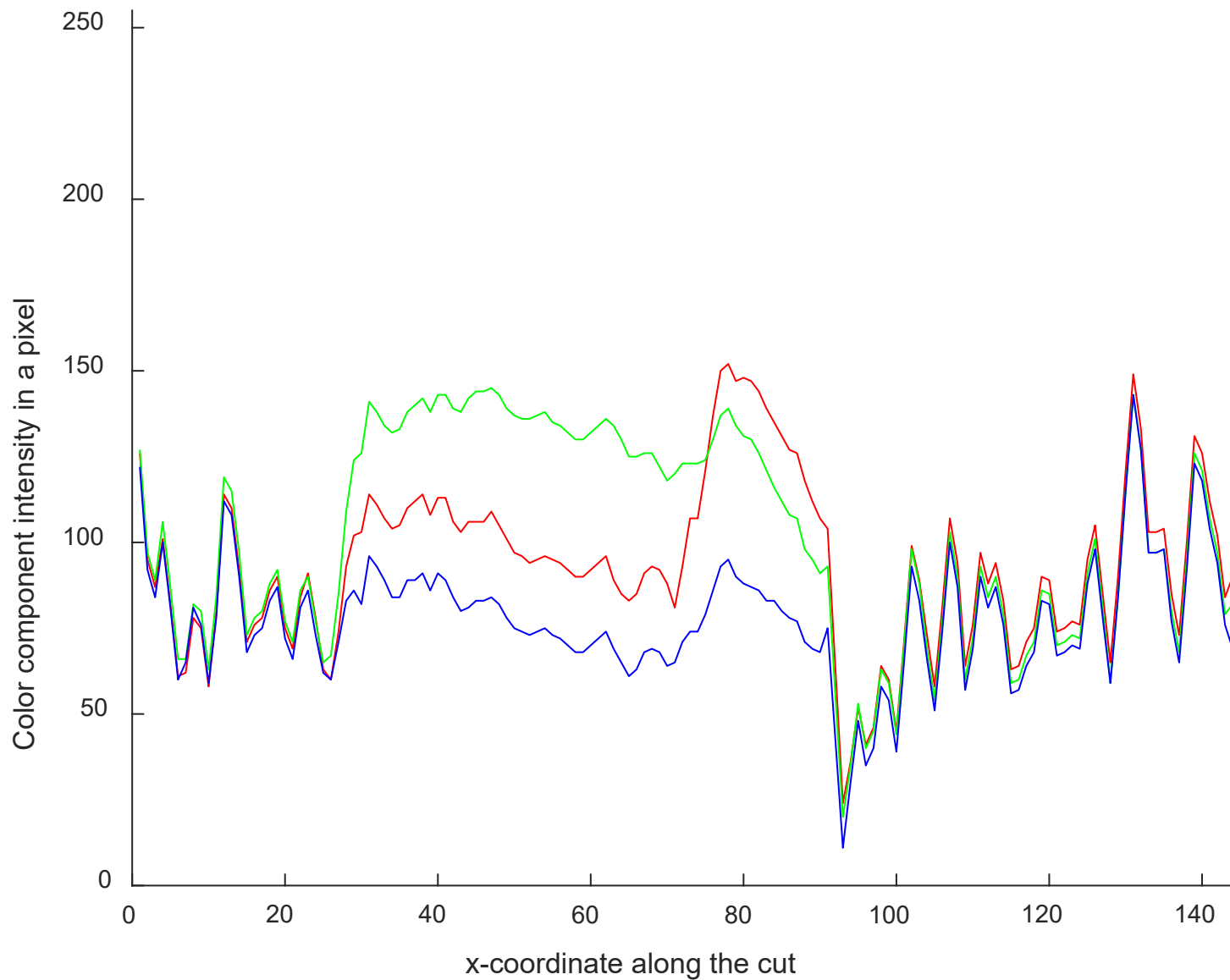
Stronger sharpening



Input color image



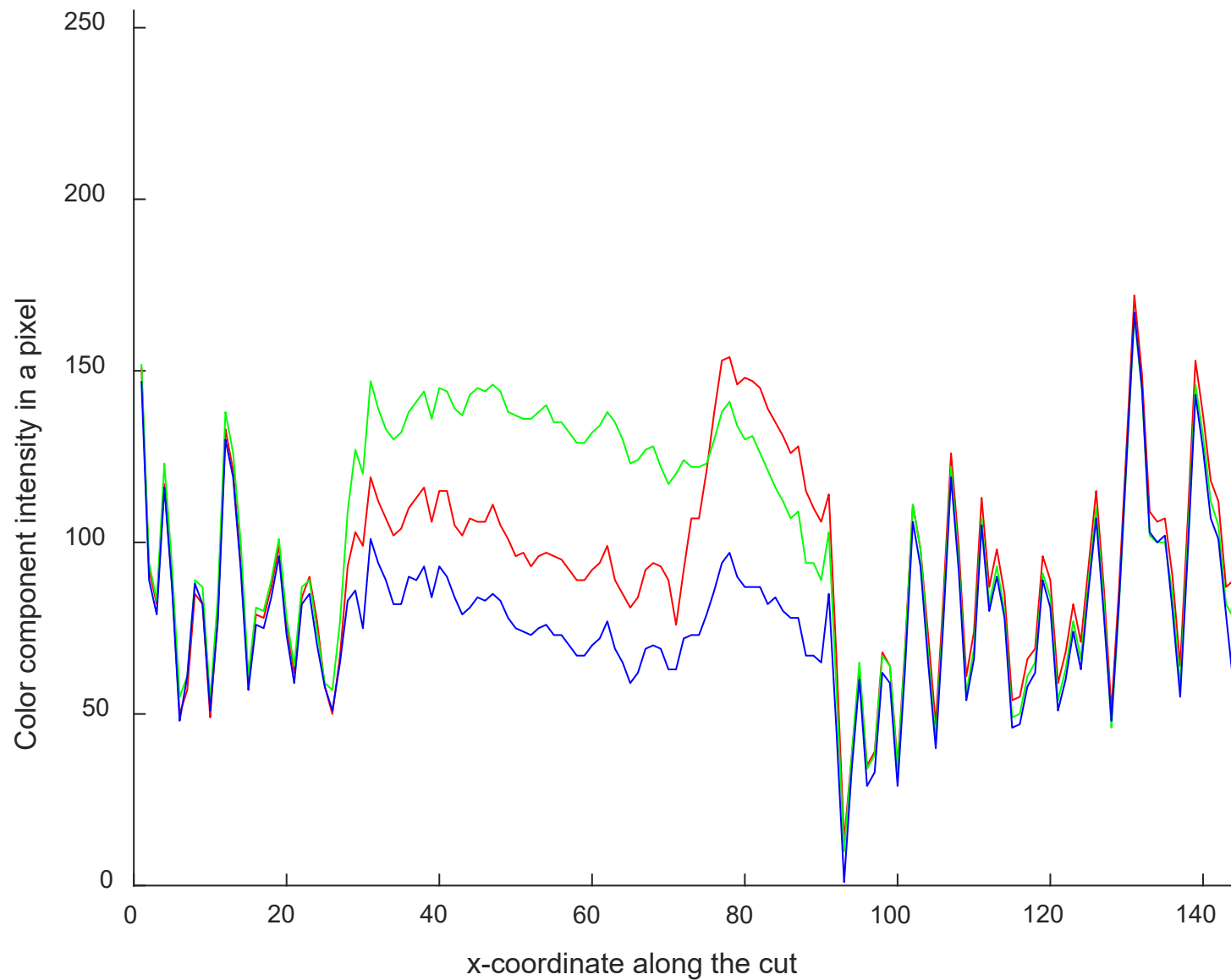
Intensity profile along a straight line segment



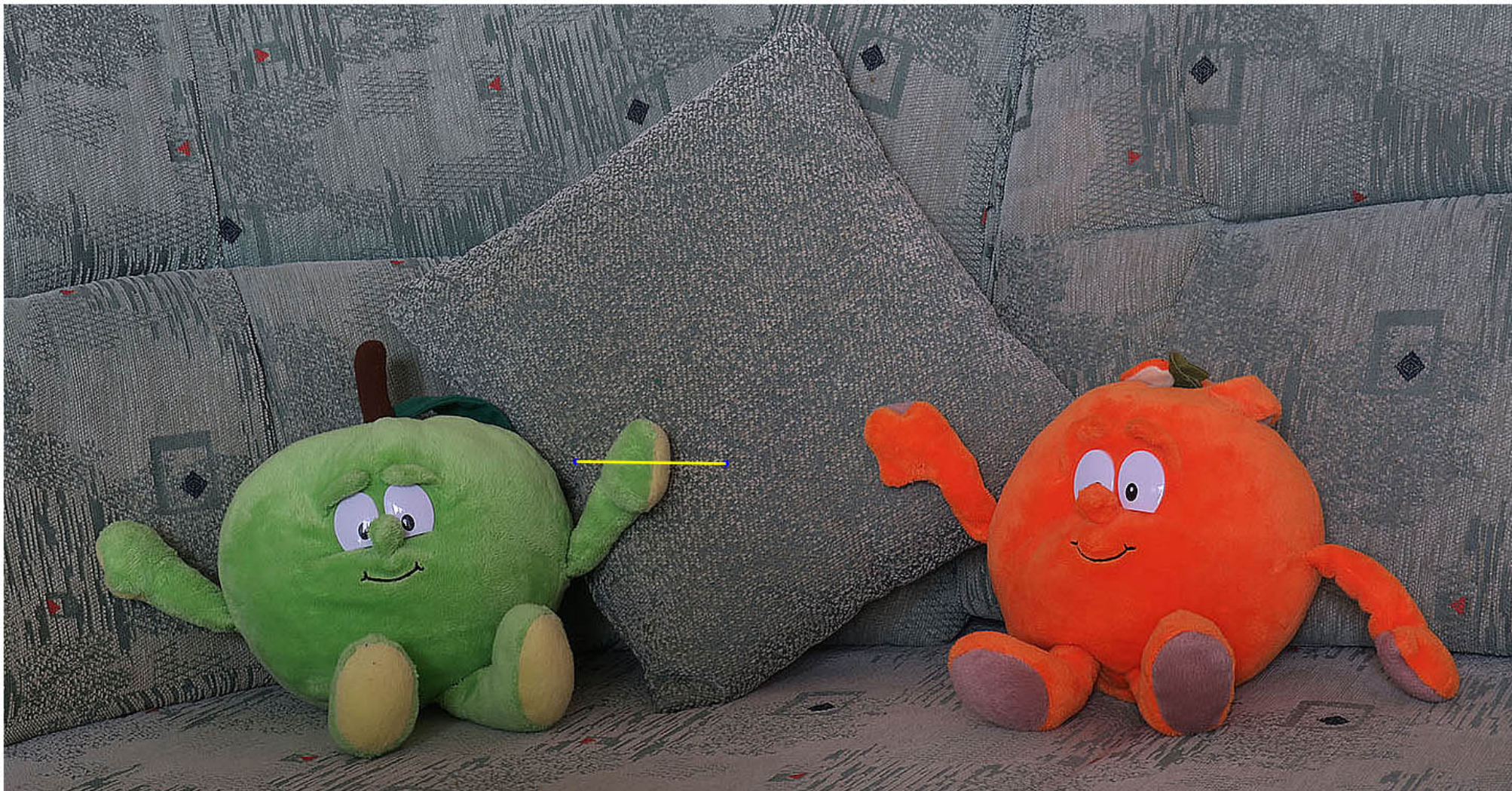
Sharpened image



Intensity profile along a straight line segment



More sharpened image



Intensity profile along a straight line segment

