ROBOT KINEMATICS

Václav Hlaváč

Czech Technical University, Faculty of Electrical Engineering
Department of Cybernetics, Center for Machine Perception
121 35 Praha 2, Karlovo nám. 13, Czech Republic

hlavac@fel.cvut.cz, http://cmp.felk.cvut.cz

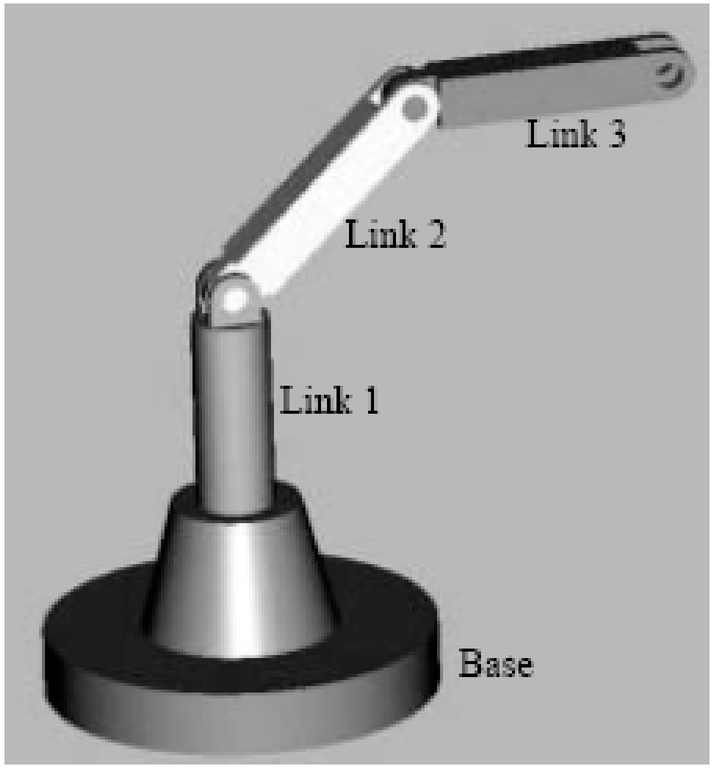
LECTURE PLAN

1. Kinematics, what is?
2. Open, closed kinematic mechanisms.
3. Sequence of joint transformations (matrix multiplications)
4. Direct vs. inverse kinematic task.

KINEMATICS

- KINEMATICS – the analytical study of the geometry of motion of a mechanism:
  - with respect to a fixed reference co-ordinate system,
  - without regard to the forces or moments that cause the motion.
- In order to control and programme a robot we must have knowledge of both its spatial arrangement and a means of reference to the environment.

OPEN CHAIN MANIPULATOR KINEMATICS



- Mechanics of a manipulator can be represented as a kinematic chain of rigid bodies (links) connected by revolute or prismatic joints.
- One end of the chain is constrained to a base, while an end effector is mounted to the other end of the chain.
- The resulting motion is obtained by composition of the elementary motions of each link with respect to the previous one.

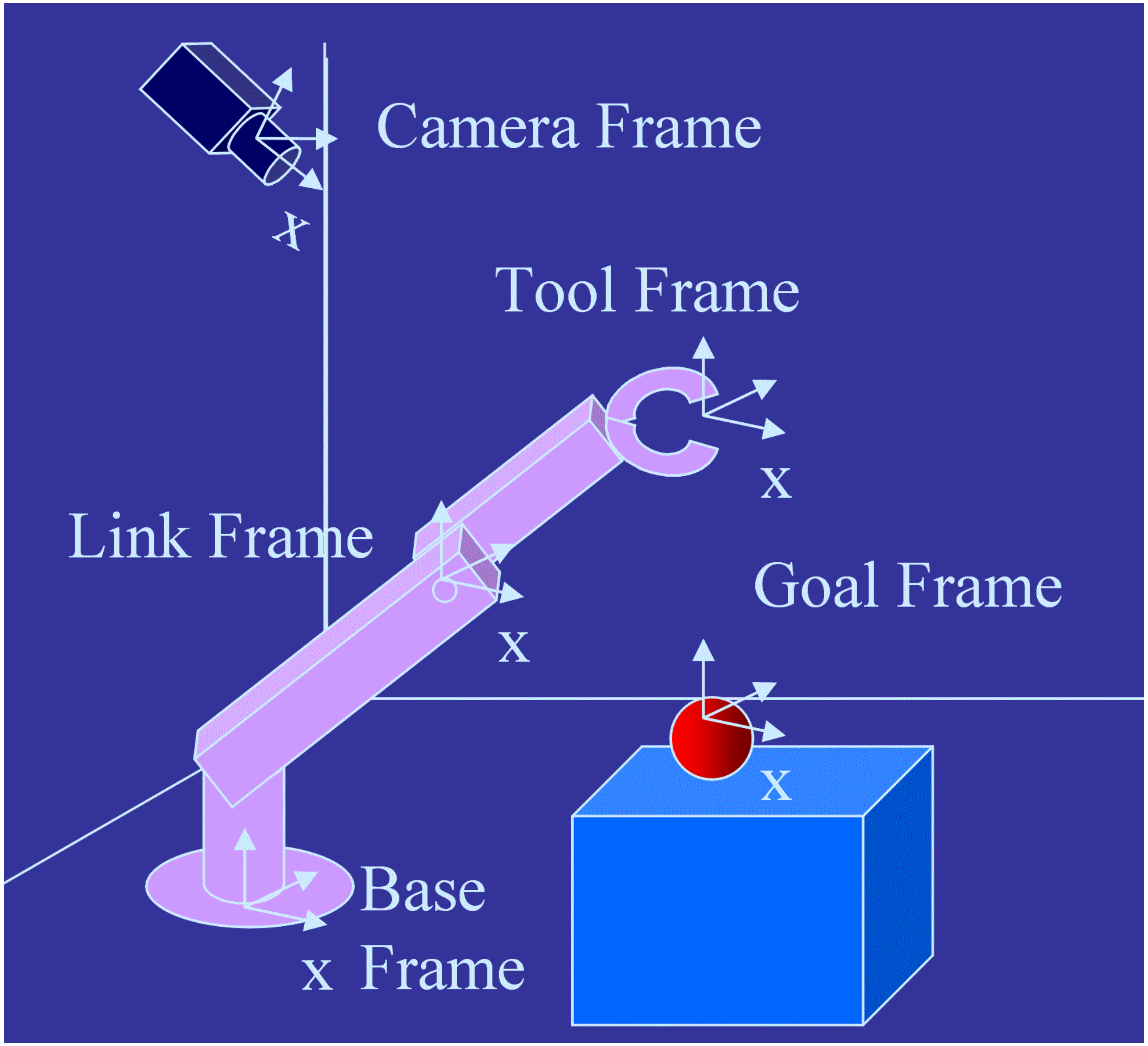
CLOSED KINEMATIC CHAIN

- Much more difficult.
- Even analysis has to take into account statics, constraints from other links, etc.
- Synthesis of closed kinematic mechanisms is very difficult.

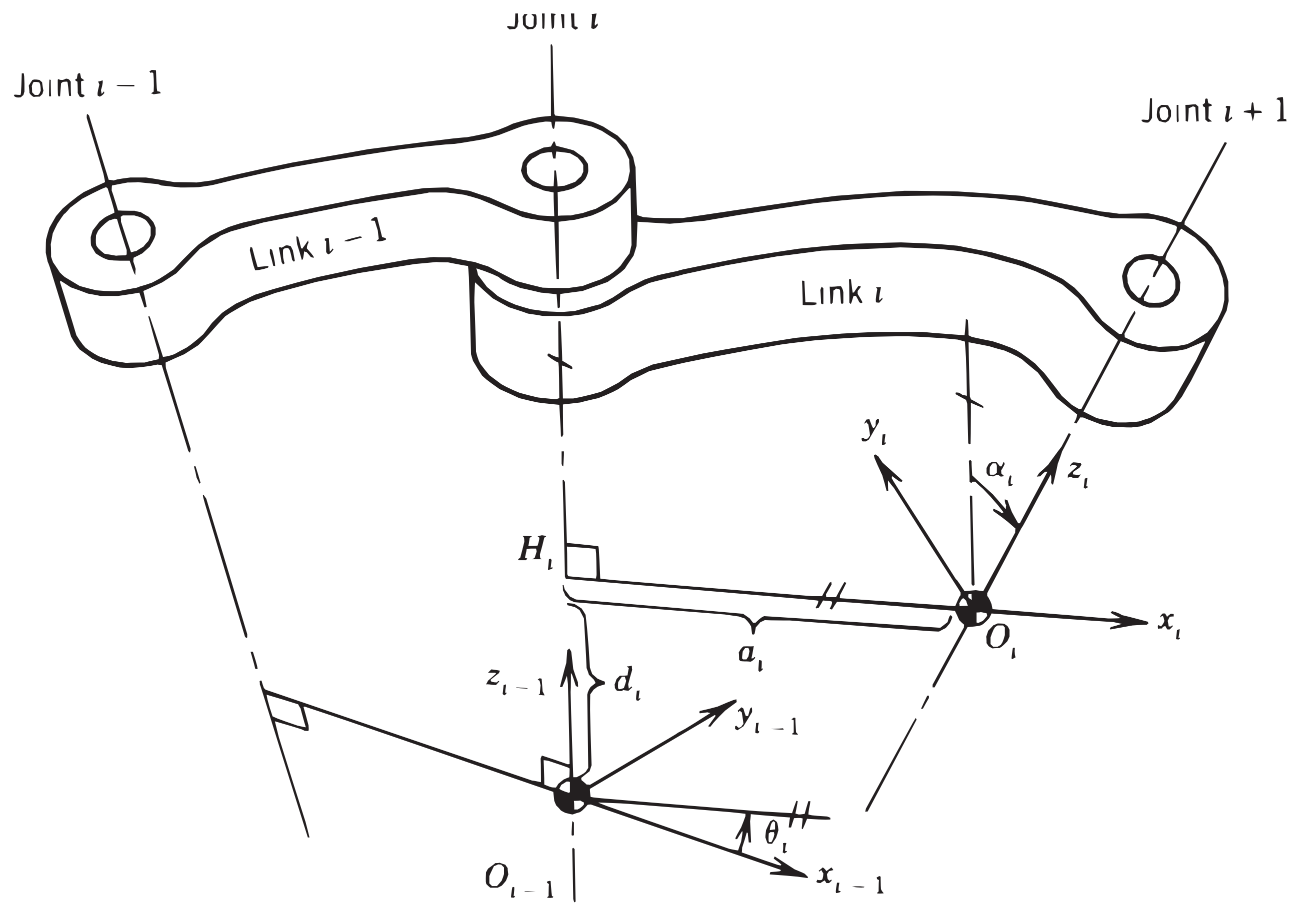
KINEMATICS vs. DIFFERENTIAL KINEMATICS

- Kinematics describes the analytical relationship between the joint positions and the end-effector position and orientation.
- Differential kinematics describes the analytical relationship between the joint motion and the end-effector motion in terms of velocities.

COORDINATE FRAMES



TWO FRAMES KINEMATIC RELATIONSHIP



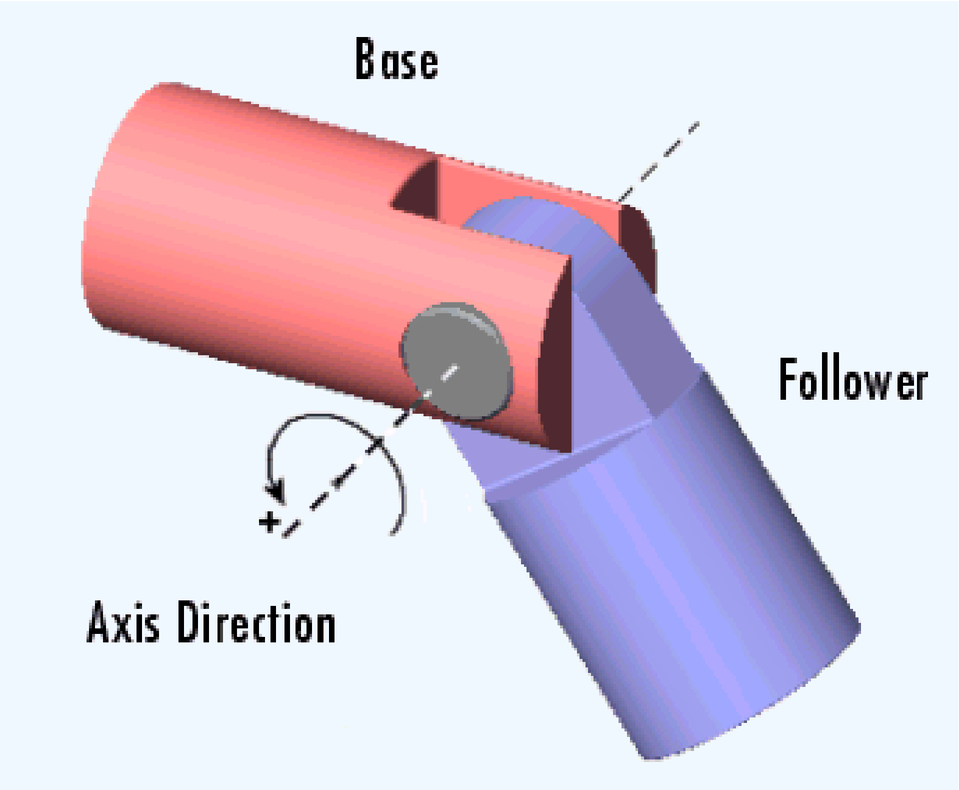
- There is a kinematic relationship between two frames, basically a translation and a rotation.
- This relationship is represented by a 4×4 homogeneous transformation matrix.

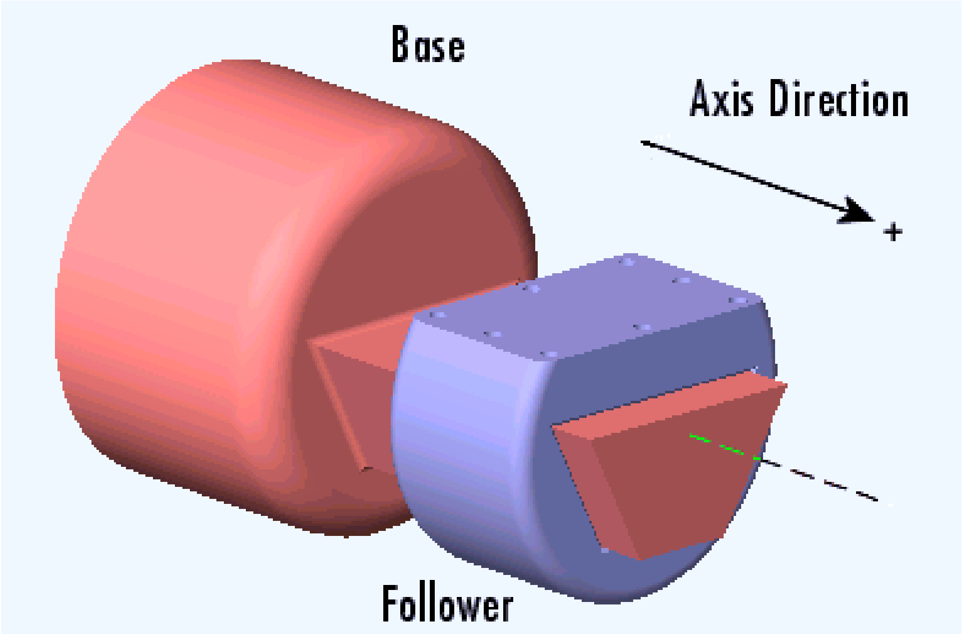
HOMOGENEOUS TRANSFORMATION

	r_1	r_2	r_3	Δx	
3x3 rotation matrix	r_4	r_5	r_6	Δy	3x1 translation
	r_7	r_8	r_9	Δz	
1x3 perspective	0	0	0	1	global scale

Rotation matrix R is orthogonal $\Leftrightarrow R^\top R = I \Rightarrow$ 3 independent entries, e.g., Euler angles.

TWO BASIC JOINTS

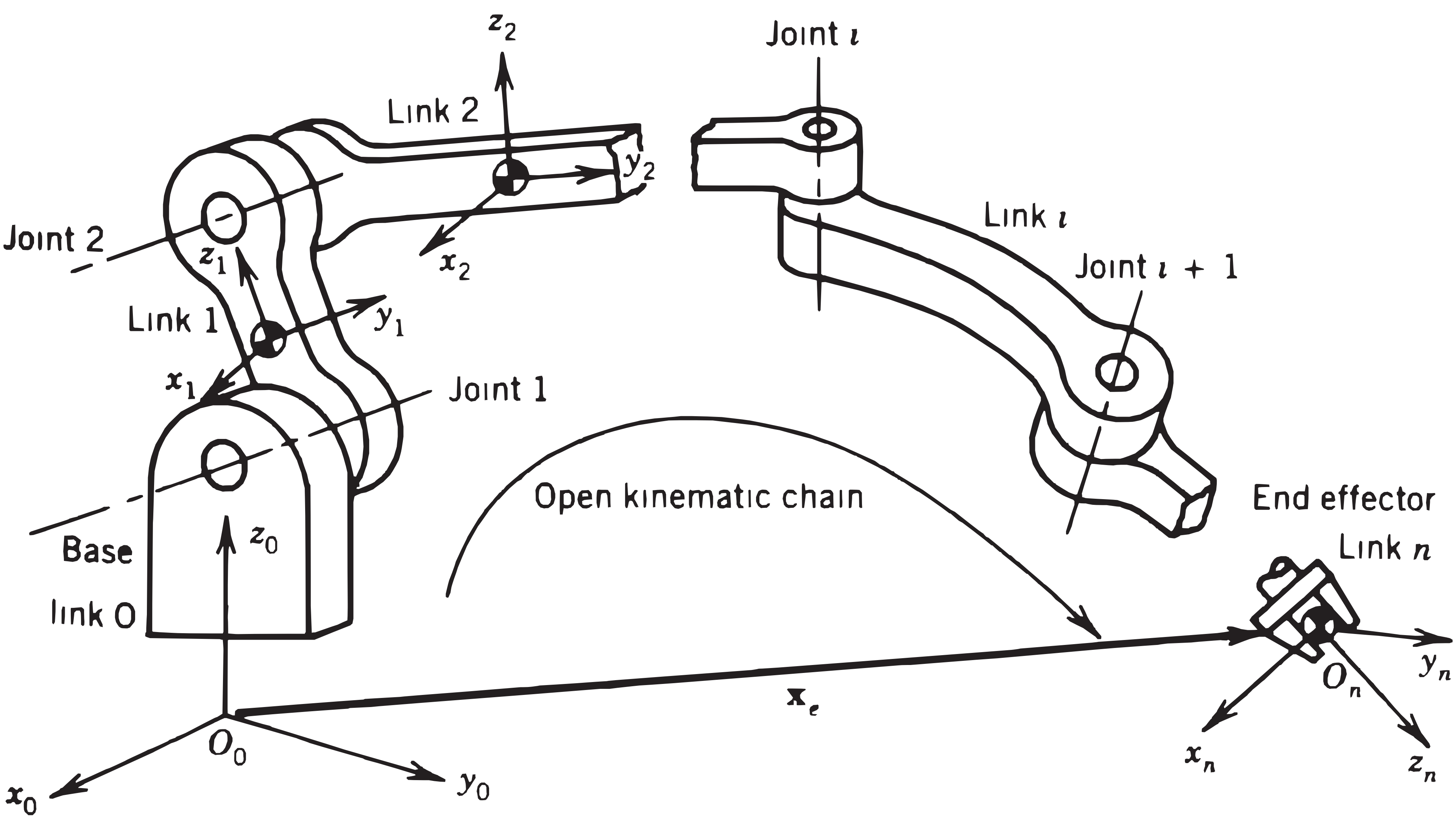




Revolute

Prismatic

OPEN KINEMATIC CHAIN



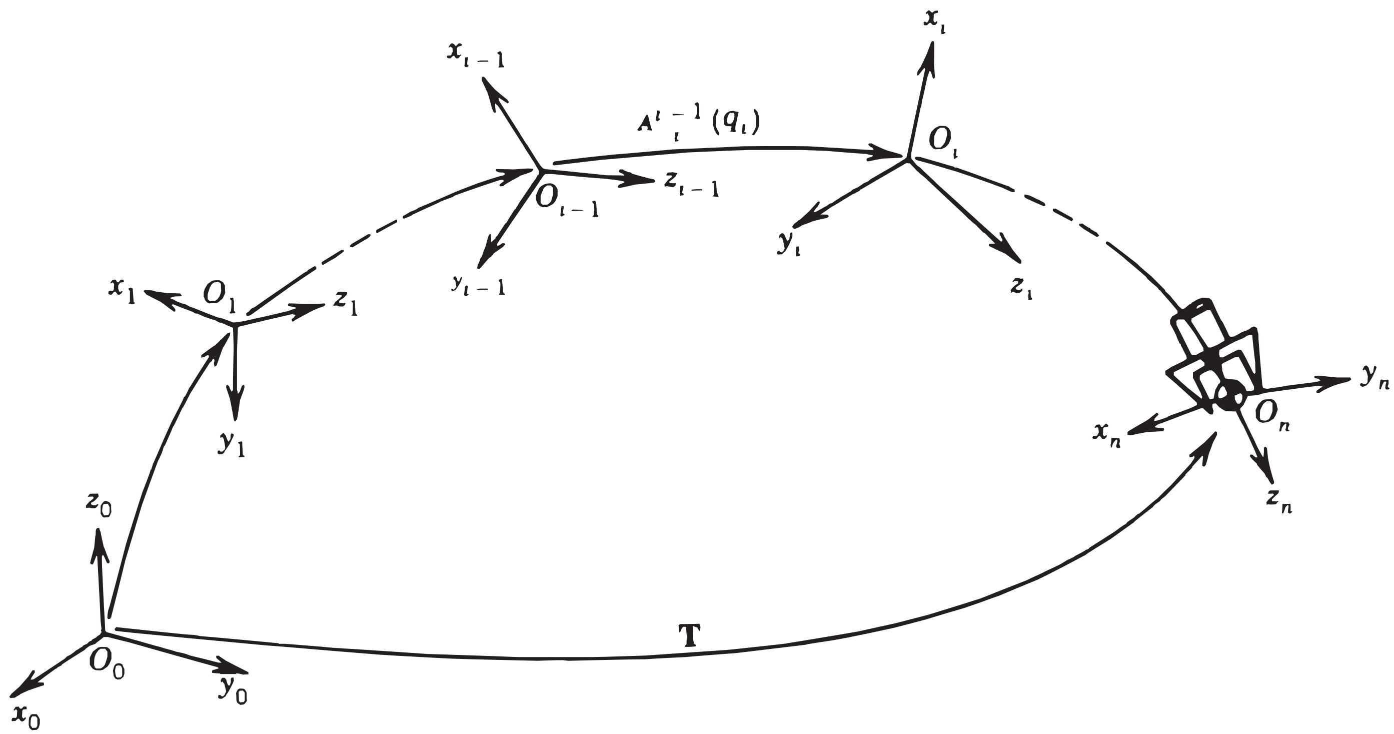
DIRECT vs. INVERSE KINEMATICS

In manipulator robotics, there are two kinematic tasks:

Direct (also forward) kinematics – Given are joint relations (rotations, translations) for the robot arm. Task: What is the orientation and position of the end effector?

Inverse kinematics – Given is desired end effector position and orientation. Task: What are the joint rotations and orientations to achieve this?

DIRECT KINEMATICS



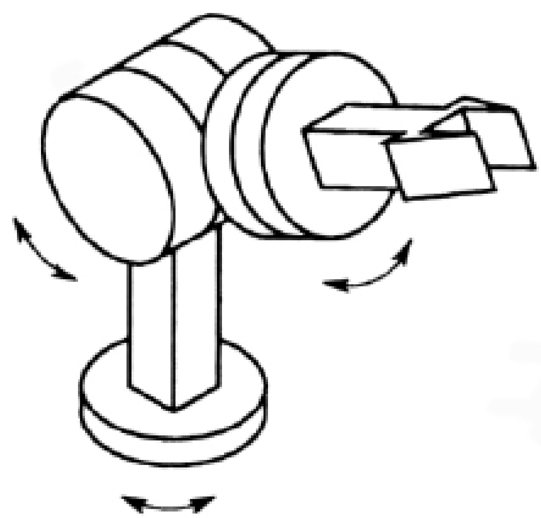
- One joint: $\mathbf{x}_i = A\mathbf{x}_{i-1}$.
- Chain of joints: $\mathbf{x}_{n-1} = A_{n-1}\,A_{n-2}\,\ldots\,A_1\,A_0\,\mathbf{x}_0$.
- Easy to compute (matrix multiplication).
- Unique solution.

INVERSE KINEMATICS

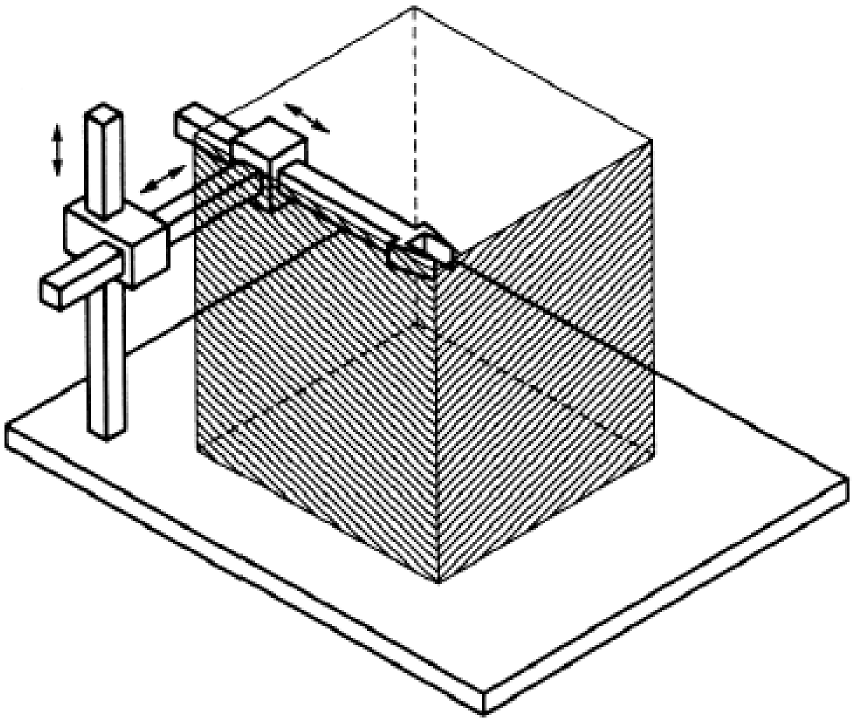
- For a kinematic mechanism, the inverse kinematic problem is difficult to solve.
- The robot controller must solve a set of non-linear simultaneous algebraic equations.
- Source of problems:
  - Non-linear equations ($\sin$, $\cos$ in rotation matrices).
  - The existence of multiple solutions.
  - The possible non-existence of a solution.
  - Singularities.

INVERSE KINEMATICS, SIMPLIFICATIONS

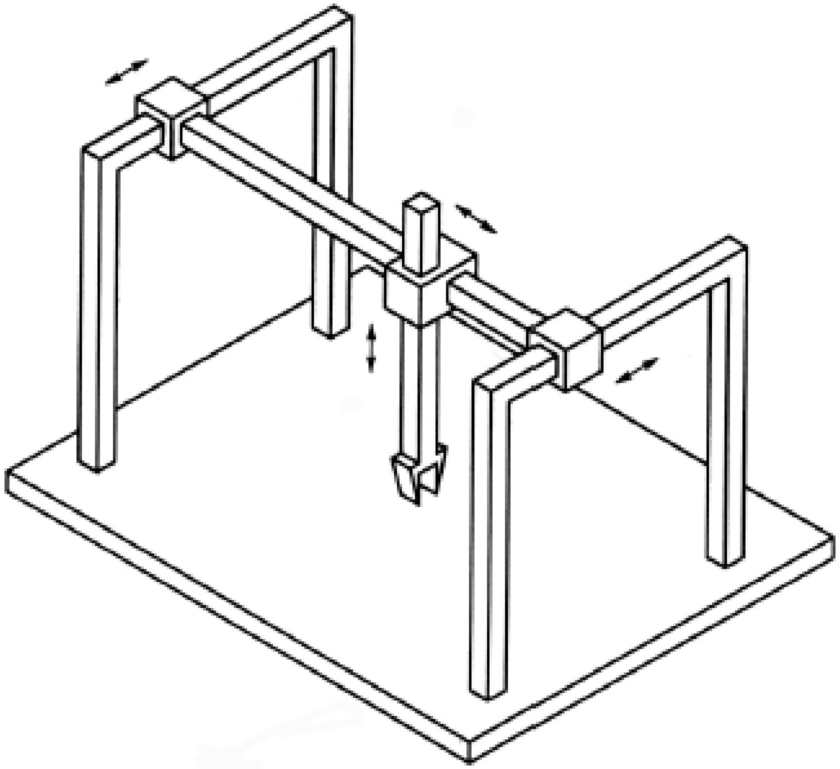
- Divide and conquer strategy. Decouple the problem into independent subproblems.
- The spherical wrist. Positioning of the wrist + positioning within the wrist.
- Design conventions, e.g. Denavit-Hartenberg systematic frame assignment.



MANIPULATOR KINEMATIC (1)

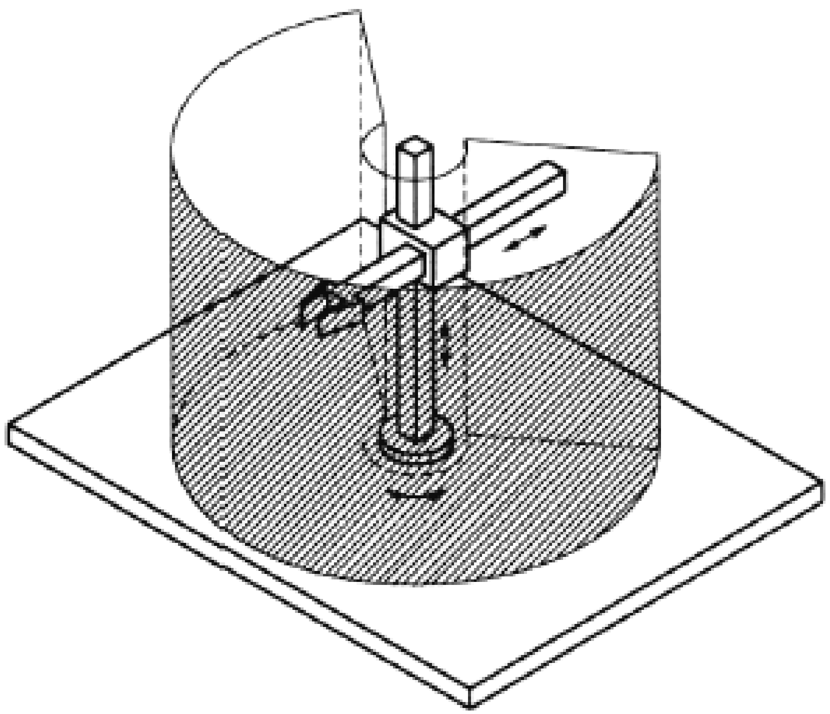


Cartesian

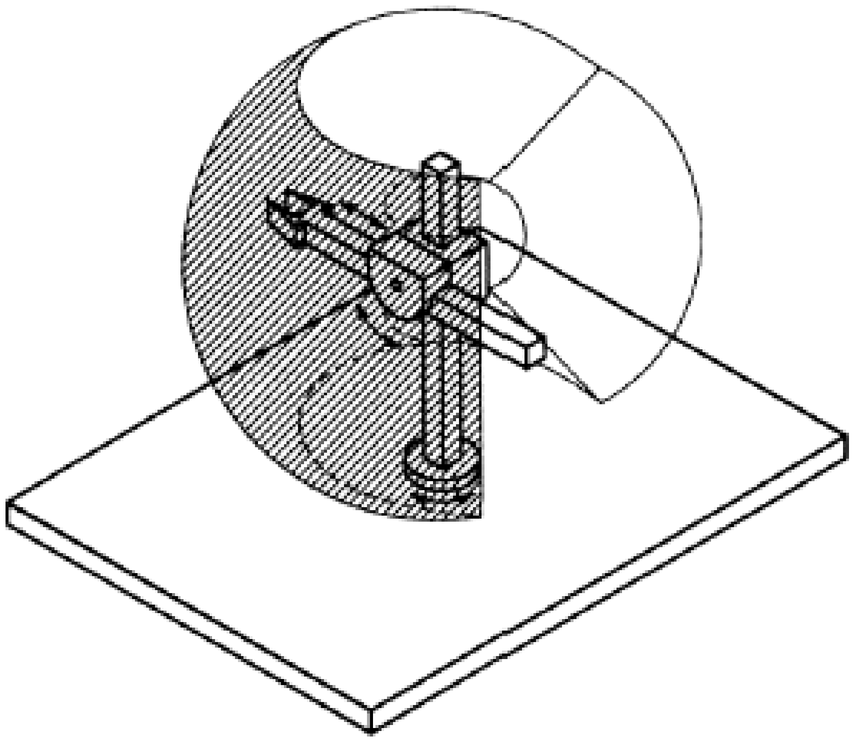


Gantry

MANIPULATOR KINEMATIC (2)

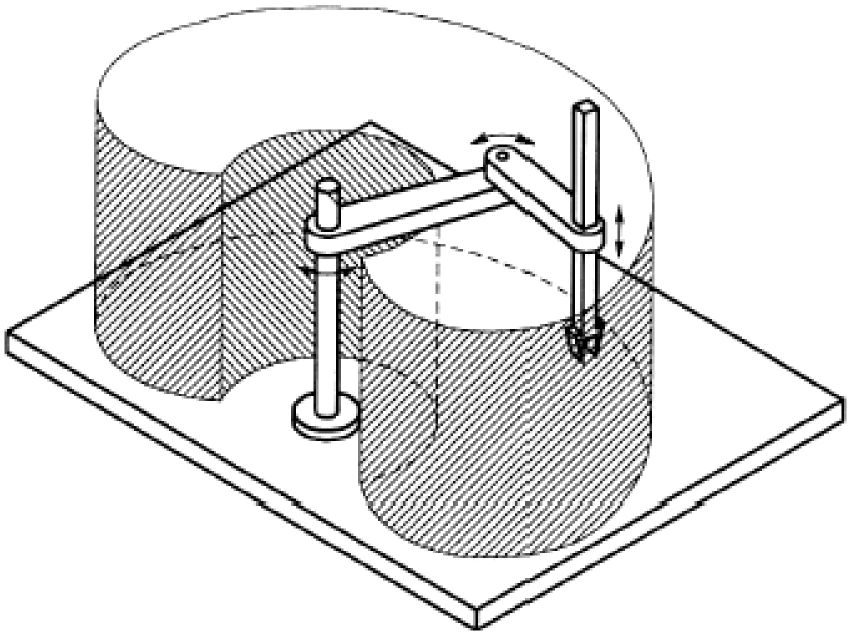


Cylindrical

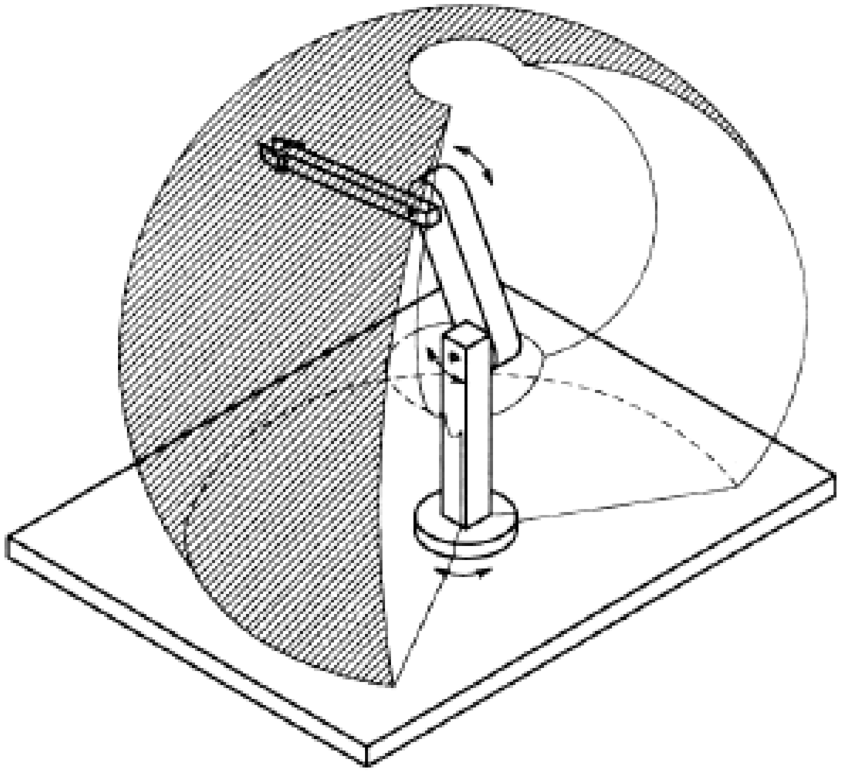


Sphere

MANIPULATOR KINEMATIC (3)



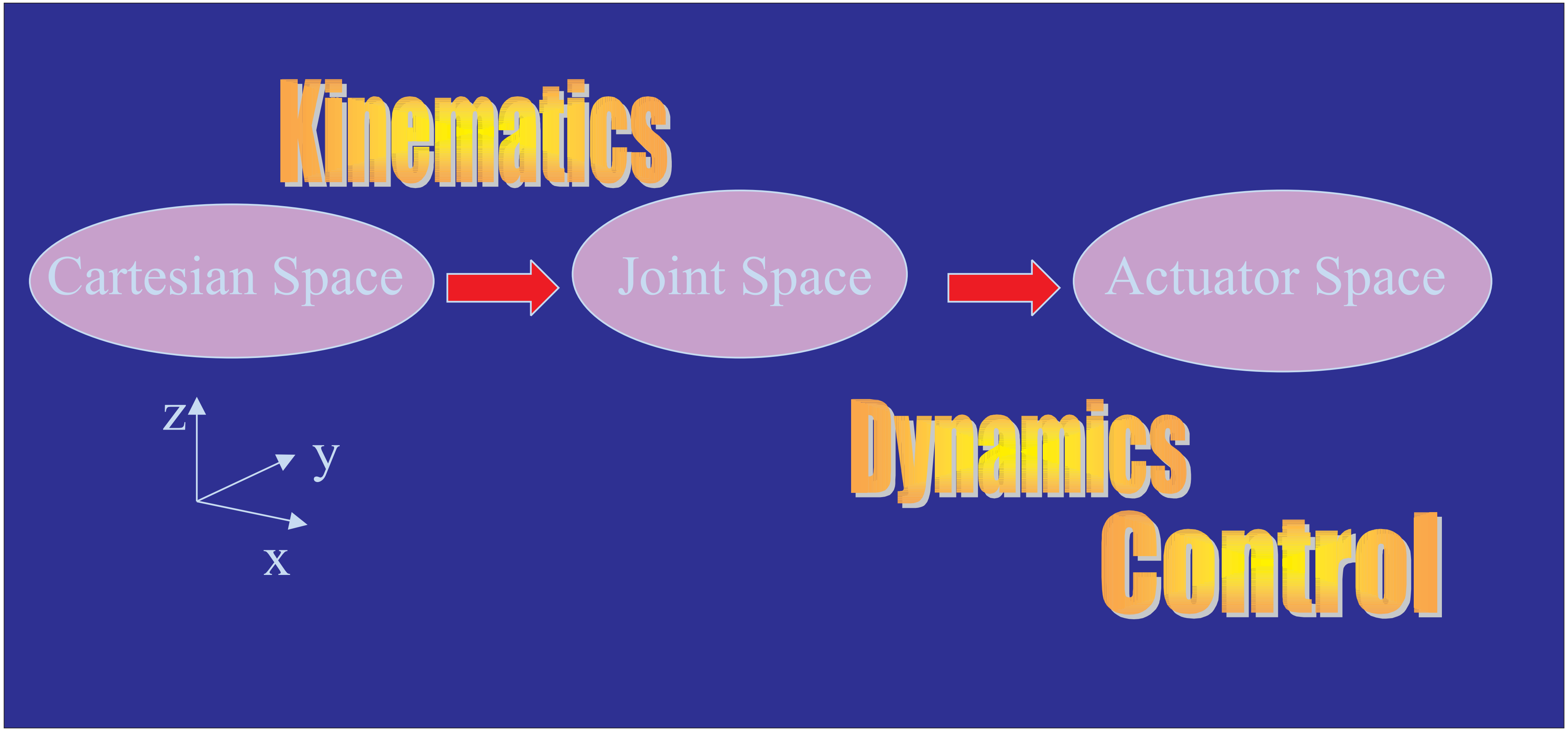
SCARA



Anthropomorphic

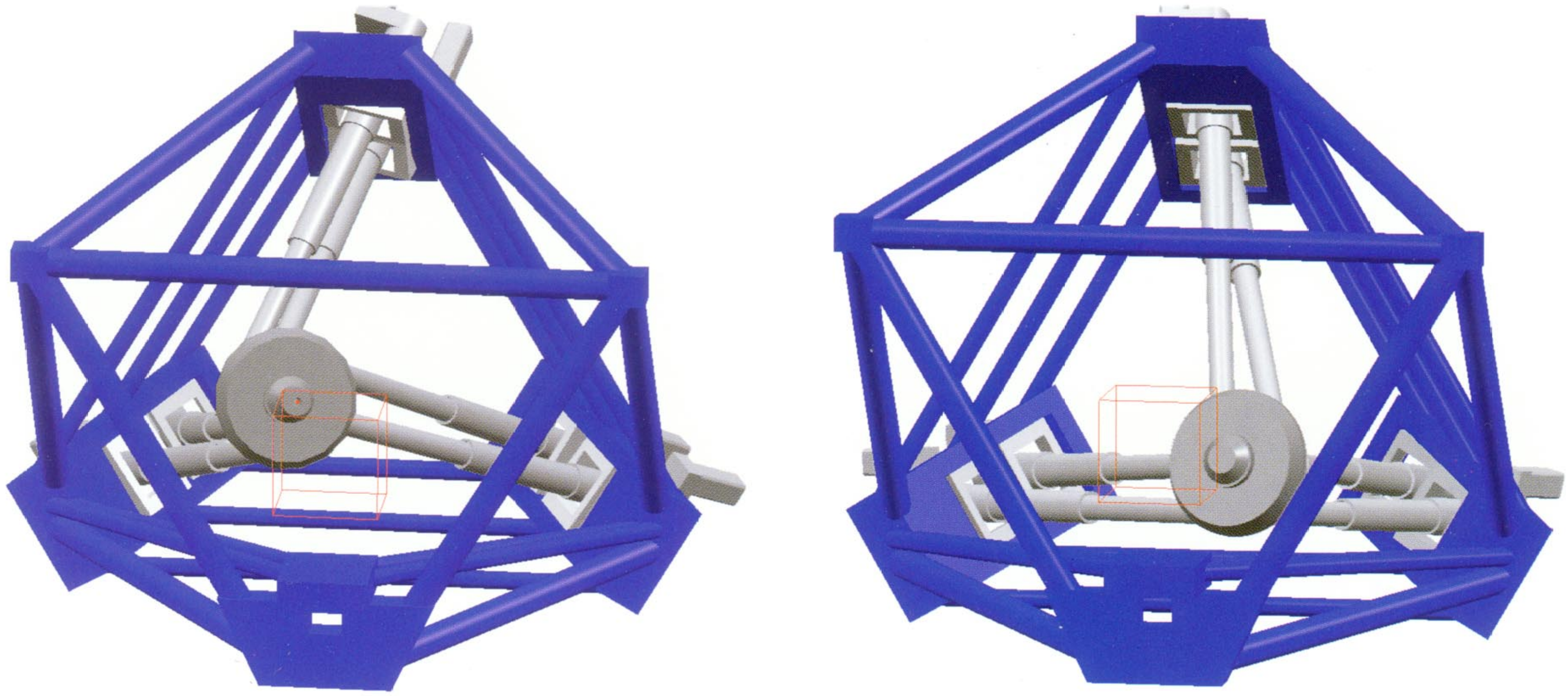
KINEMATICS → DYNAMICS, CONTROL

Kinematics is only the first step towards robot control !

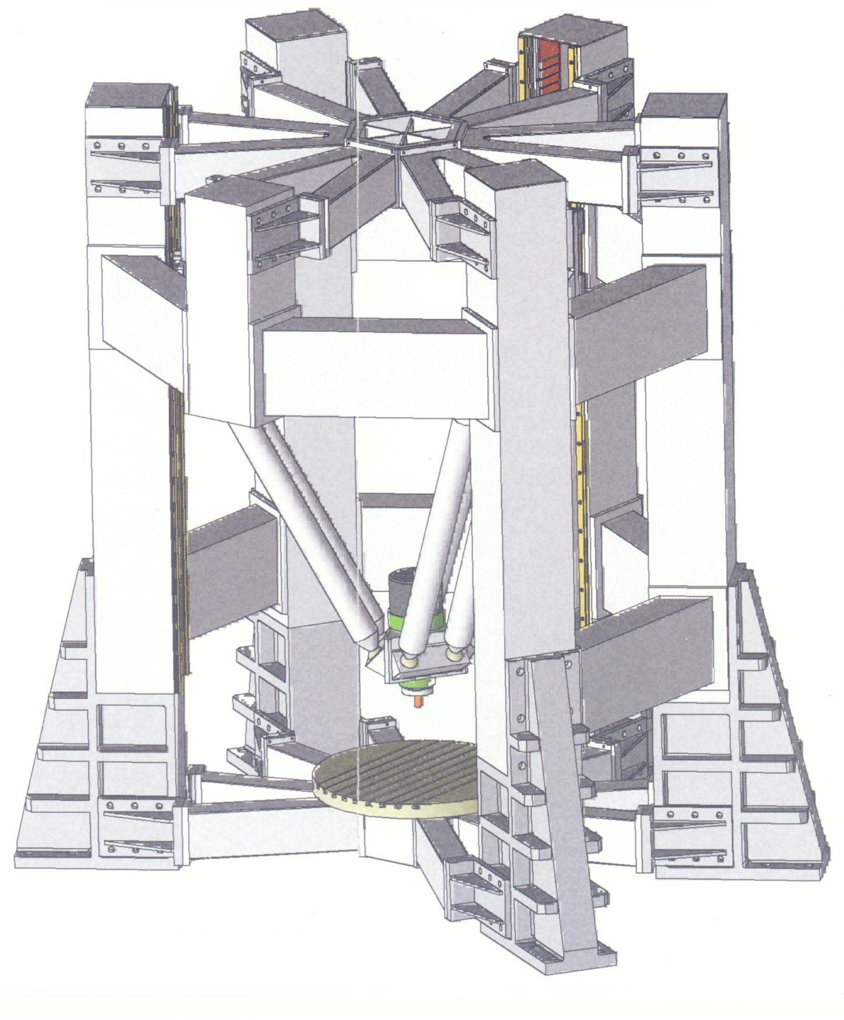


CLOSED PARALLEL CHAIN

Hexamod



REAL HEXAMOD (1)



REAL HEXAMOD (2)