Camera Uncertainty Computation in Large 3D Reconstruction

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KEY KONCEPTS

- **3D scene** – a set of the parameters (e.g. camera poses and 3D points) with relationships defined by residual function
- **Uncertainty** – can be approximated by normal distribution and described by a covariance matrix
- **Uncertainty propagation** – in our case, the transport of the uncertainties of feature points to the parameters of 3D scene

UNCERTAINTY PROPAGATION

- The relationships between parameters of 3D scene and feature points is described by residual function

$$r(\theta) = \|f(\theta) - \mathbf{u}\|$$

θ … parameters of the 3D scene
$\mathbf{u}$ … feature points
$\boldsymbol{f}$ … projection functions

- The projection function is usually linearized by

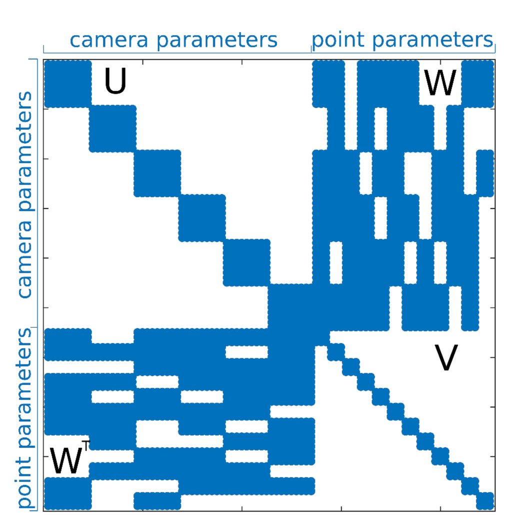
$$f(\theta) \approx f(E(\theta)) + J(E(\theta) - \theta)$$

- The residuals do not change if we scale, shift or rotate whole 3D scene => in this direction has J null space and the uncertainty is infinite
- [Kanatani_2001] defined the normal form of the covariance matrix which has zero uncertainty in the direction of null space of J

$$\Sigma_\theta = (J^T \Sigma_{\mathbf{u}} J)^+ = \begin{pmatrix} U_N & W_N \\ W_N^T & V_N \end{pmatrix}^+$$

$U_N \in \mathbb{R}^{np \times np}$ $V_N \in \mathbb{R}^{3m \times 3m}$

$\Sigma_{\mathbf{u}}$ … covariances of feature points
n … number of cameras
p … number of camera parameters
m … number of points in 3D



MOTIVATION

- **Precisely computed uncertainty of parameters of 3D scene**
- **Robust SfM** – filtering the most unconstrained cameras, i.e. do not extend partial reconstruction about highly unconstrained camera
- **Speed up the SfM** – reduce the number of optimized parameters of a scene

DECOMPOSITION

- Because of the inversion instead of Moore-Penrose pseudoinversion we could decompose the Fisher information matrix

$$\Sigma_\theta = \sigma^2 R \left(\begin{bmatrix} I & -Y^T \\ & I \end{bmatrix} \begin{bmatrix} Z & 0 \\ 0 & V \end{bmatrix} \begin{bmatrix} I & 0 \\ -Y & I \end{bmatrix} \right)^{-1} R^T$$

$Y = -V^{-1} W^T$ … definition for brevity
$Z = U + WY$ … Schur complement matrix $\in \mathbb{R}^{np \times np}$

- The problem is that Z is rank deficient for large scenes and the decomposition do not hold for Moore-Penrose pseudoinversion of Z

APPROX. OF Z^{-1}

- We define the function g based on λ

$$g(\lambda) = (Z + \lambda I)^{-1}$$

- and estimate $g(0)$

$$g(0) = (Z + \lambda I)^{-1} \sum_{t=1}^{\infty} \frac{\lambda^t}{(t-1)!} (Z + \lambda I)^{-(t+1)}$$

REGULARISATION

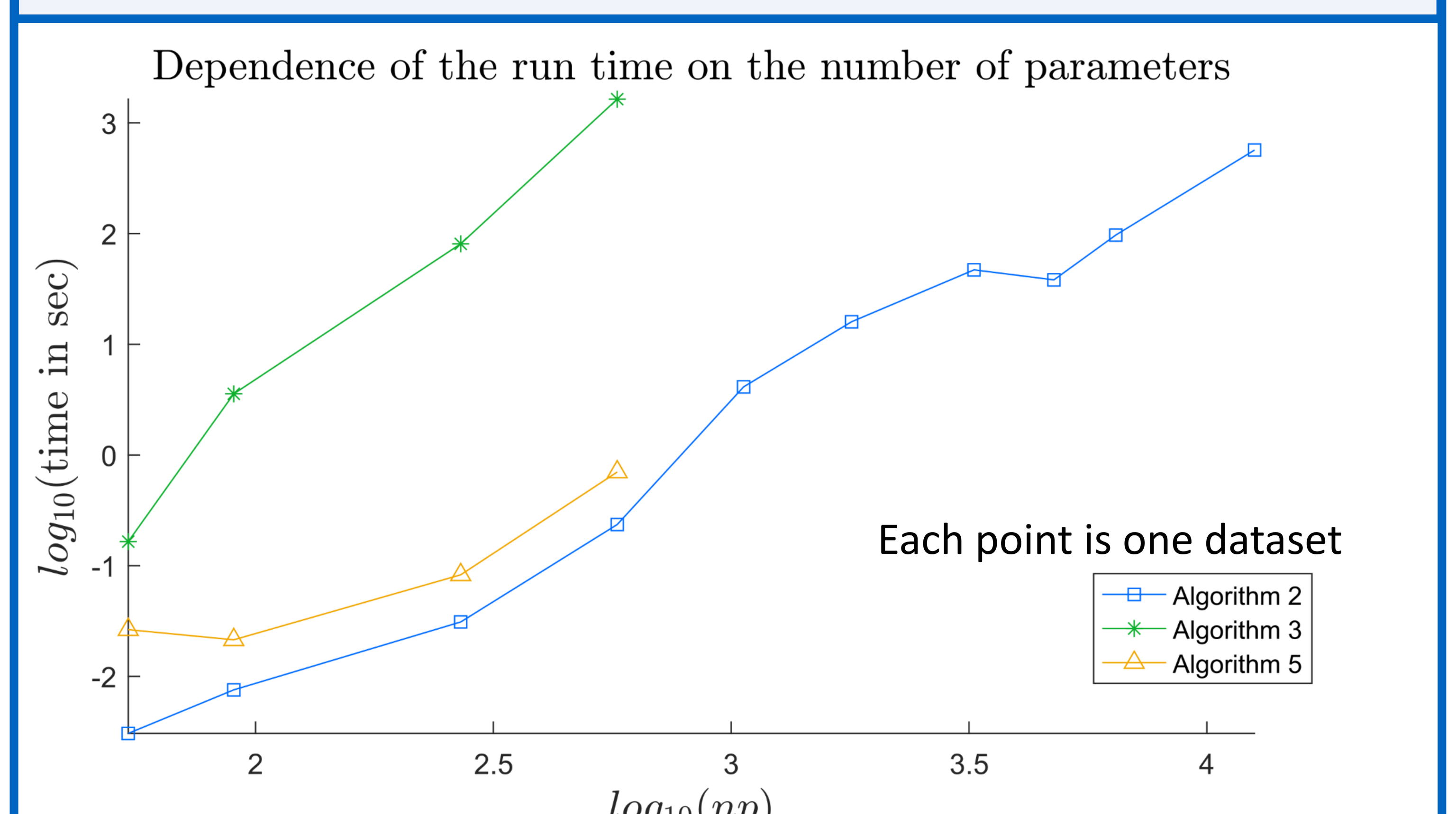
- The Jacobian J contains large range of values (i.e. some parameter, e.g. angle axis, contains small values and influence the residuals much more than higher changes of higher values, e.g. coordinates of 3D point)
- We regularize the problem by matrix R

$$\Sigma_\theta = R (R^T J^T \Sigma_{\mathbf{u}} J R)^{-1} R^T = \begin{pmatrix} U & W \\ W^T & V \end{pmatrix}^{-1}$$

$R = R_p R_s$ R_p … projection to the set of essential parameters
R_s … scale of the columns of J to have its norms equal 1

- The matrix R_p was empirically found such a way to minimize the difference of the inversion instead of Moore-Penrose pseudoinversion
- R_p fix three most distant points found by RANSAC

SPEED



- 1. alg. ≈ 22h for Daliborka • speed of 2. alg. ≈ 4. alg. ≈ 7. alg.
- 3. alg. and 5. alg. cannot be evaluated (memory requirements)

PRECISION