Boolean Leximax Optimisation using Iterative SAT Solving

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SAT 2022

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Package Upgradeability Problem

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$ apt install libreoffice
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`ure`, 7645 requires:

- `debconf` ≥ 1675 or `debconf-2.0`

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`ure`, 7645 **requires:**

- `debconf` ≥ 1675 **or** `debconf-2.0`

`ure`, 7645 **conflicts with:**

- `cli-uno-bridge` < 16229

Package Upgradeability Problem

System	Manager
ubuntu	`apt`
fedora	`dnf`
python	`pip`
OCaml	`opam`

Package Upgradeability Problem

Package Upgradeability $\rightsquigarrow$ SAT

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Minimize:

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Minimize:

- Number of **removed** packages
- Number of not **up-to-date** packages
- ...

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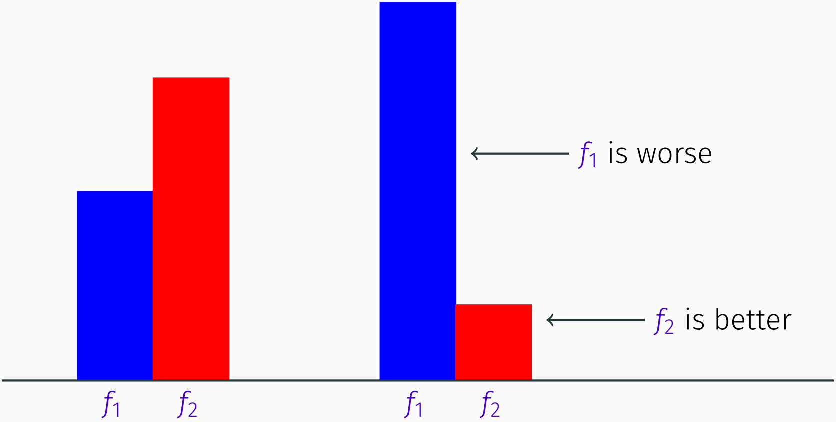
Multi-Objective MaxSAT

Hard clauses + **Multiple** sets of Soft clauses



What is the optimum?

Alice & Bob Pay a Fine

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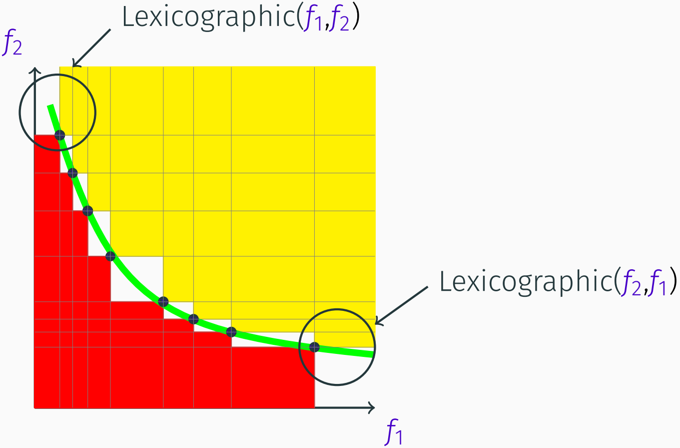
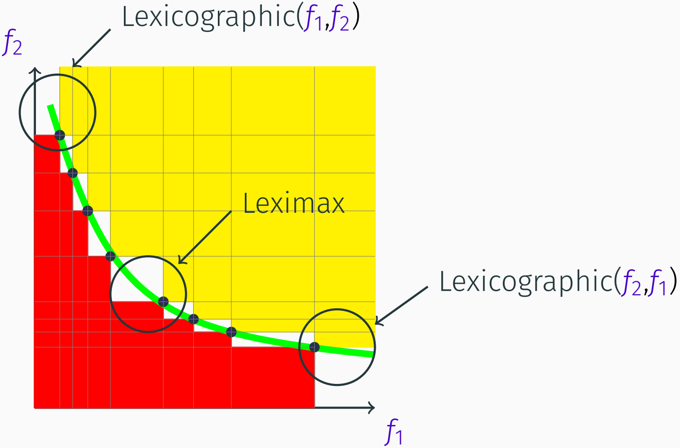
- **Generalize by**:

  Sort decreasingly and compare lexicographically

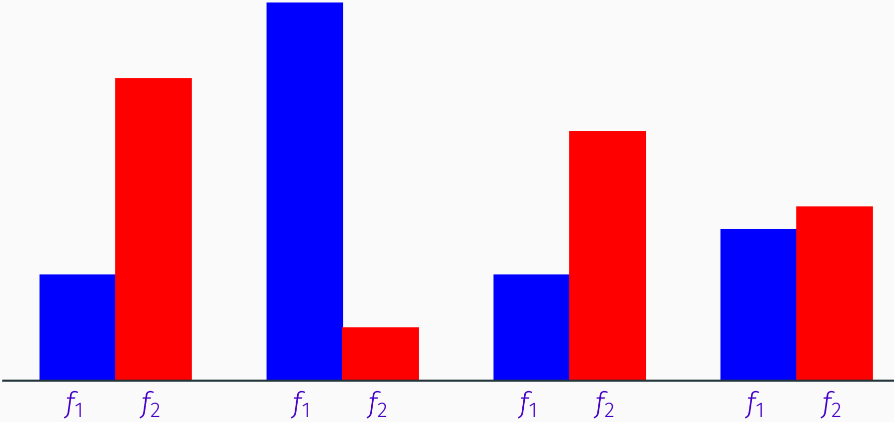
Lexicographic vs Leximax



Lexicographic vs Leximax



Leximax-optimum



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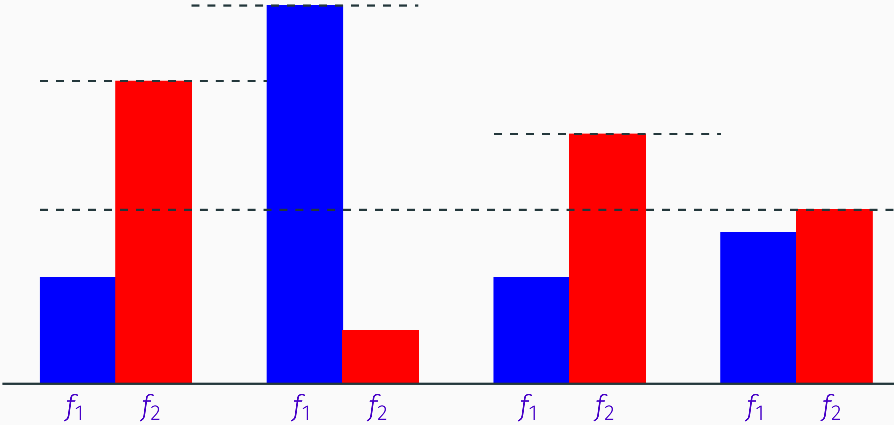

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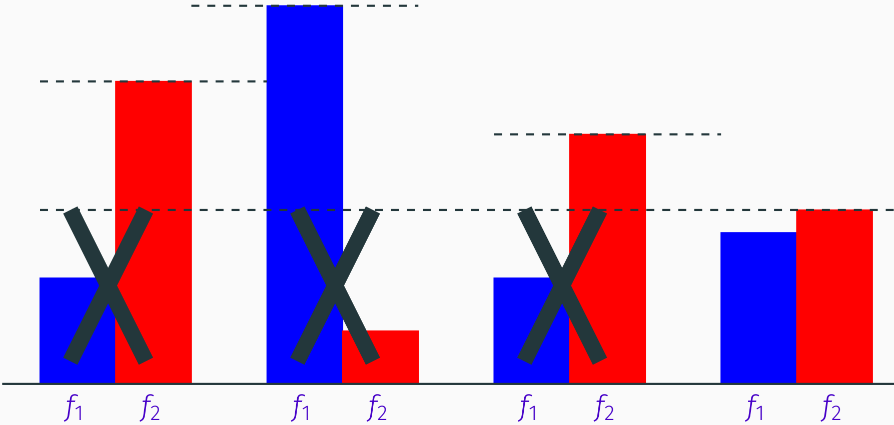

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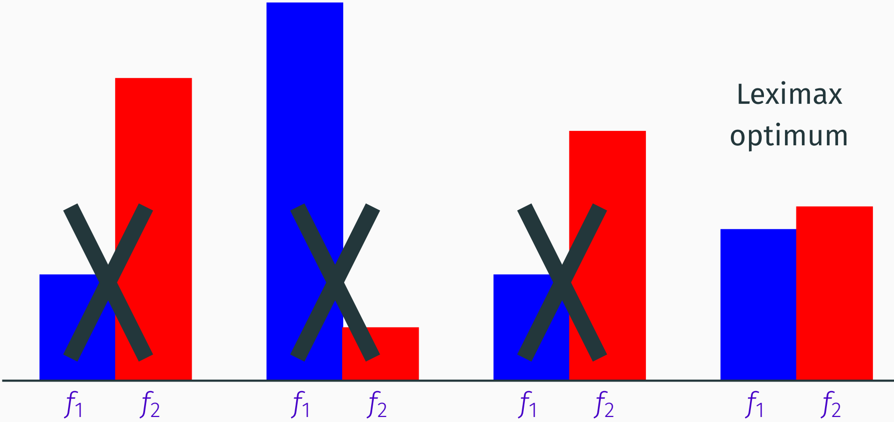

SAT-based Leximax Optimisation

$f_1 : x_1 + x_2 + x_3$

$f_2 : x_4 + x_5 + x_6$

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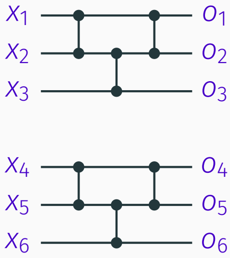

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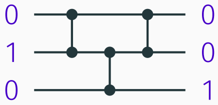

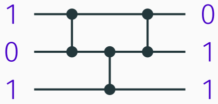

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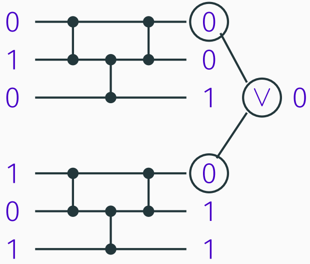

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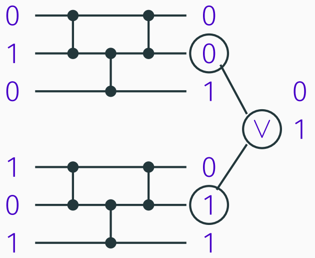

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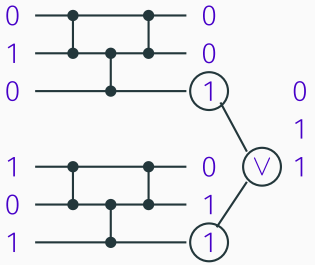

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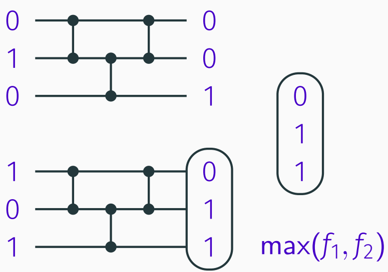

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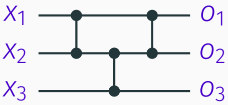

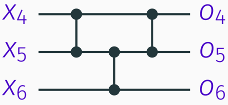

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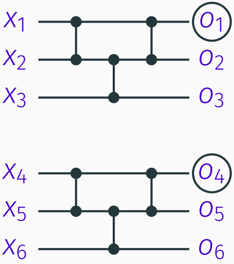

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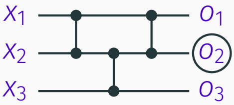

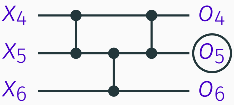

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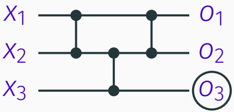

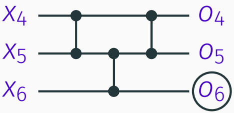

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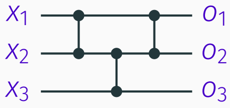

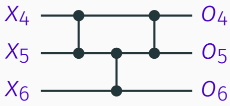

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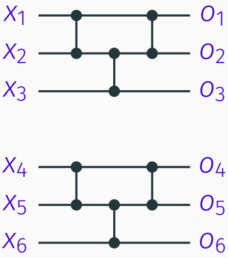

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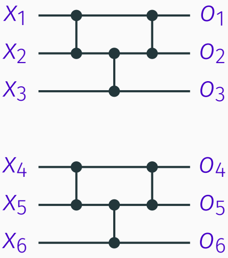

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Optimum = k

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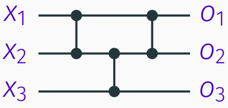

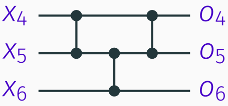

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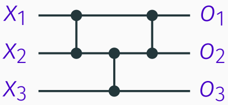

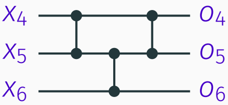

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Fix $y_1 + y_2 + y_3 = k$

Repeat with **second** maximum.

Core-guided Approach

- Major difficulty is the encoding of sorting networks

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  - Permit non-optimality if it appears in a core.
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    - Augment old sorting network with new variables
    - Or Rebuild the network.

Evaluation

- Instances:
  Package Upgradeability benchmarks
  from the Mancoosi project

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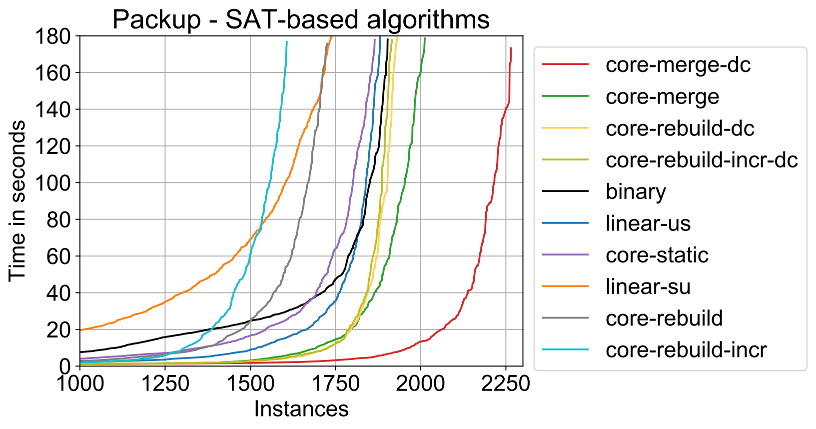
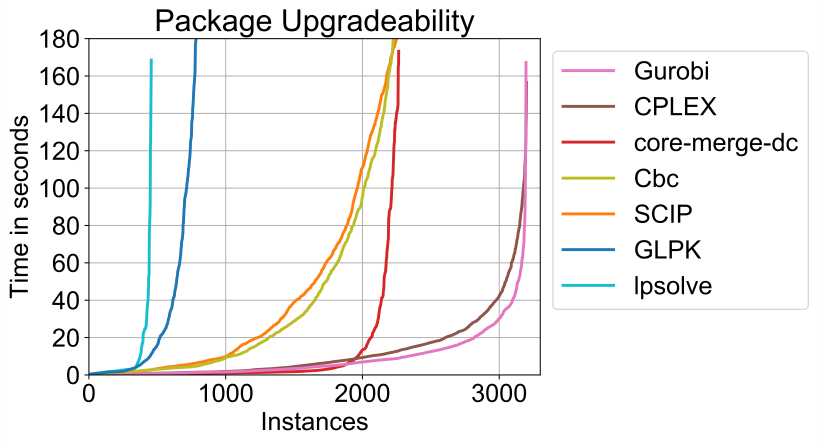
- `packup` (SAT-based algorithms) with `CaDiCaL`
  - various non-core-guided search (binary, linear, etc.)
  - various core-guided search differing on how to augment the sorting network

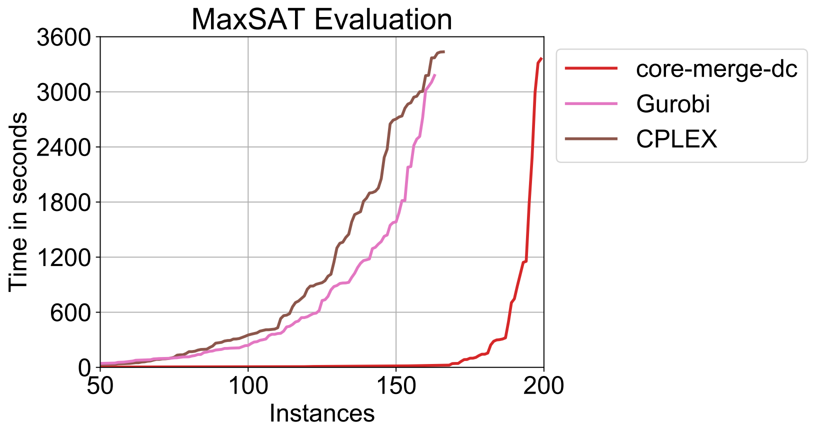
Comparison — SAT-based



Evaluation — Package Upgradeability



Evaluation — SAT Competition



Conclusions and Future Work

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- Solving Multi-Objective Optimization for **Leximax**
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- Evaluation on Package Upgradeability
- Core-guided the best out of the SAT-based