Robot kinematics

Václav Hlaváč

Czech Technical University in Prague

Czech Institute of Informatics, Robotics and Cybernetics

166 36 Prague 6, Jugoslávských partyzánů 3, Czech Republic

http://people.ciirc.cvut.cz/hlavac, vaclav.hlavac@cvut.cz

also Center for Machine Perception, http://cmp.felk.cvut.cz

Outline of the talk:

1. Kinematics, what is?
2. Open, closed kinematic mechanisms.
3. Sequence of joint transformations (matrix multiplications).
4. Direct vs. inverse kinematic task.

Initial comments

- We will refer here to a robot as a proxy for a mechanical device, its position, stiffness or dynamics is of interest.
- The terms and laws studied here can be applied to an industrial manipulator, any other robot, and any other mechanism with moving components.

Mechanics and its parts

Kinematics analyzes the geometry of a motion analytically, e.g. of a robot:

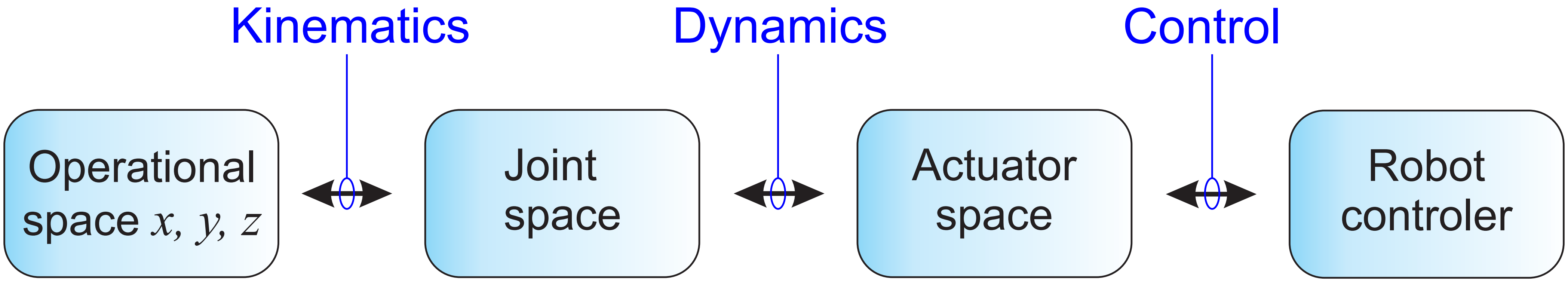
- With respect to a fixed reference co-ordinate system.
- Without regard to the forces or moments that cause the motion.
- Essential concepts are position and orientation.

Statics deals with forces and moments applied on the mechanism, which is not moving. The essential concepts used are stiffness $[Nm^{-1}]$ and stress $[Nm^{2}]$.

Dynamics analyzes forces $[N]$ and moments $[Nm]$, which result from motion and acceleration $[m\,s^{-2}]$ of the mechanism and the load.

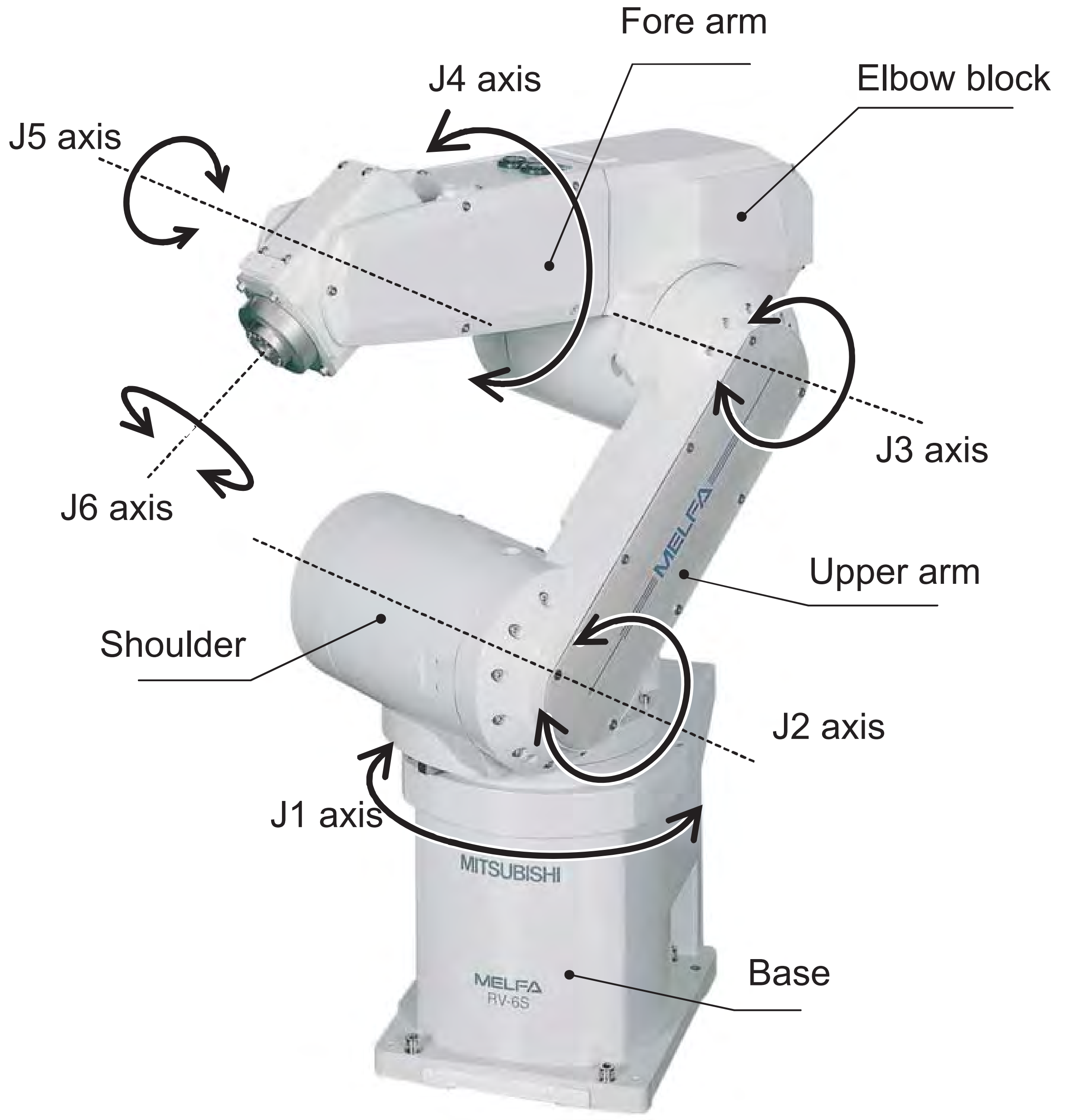
Need of kinematics in robotics

- Knowing the kinematical description of a robot is a prerequisite of its control and programming.
- Kinematics provides knowledge of both robot spatial arrangement and a means of reference to the environment.
- Kinematics is only the first step towards robot control !

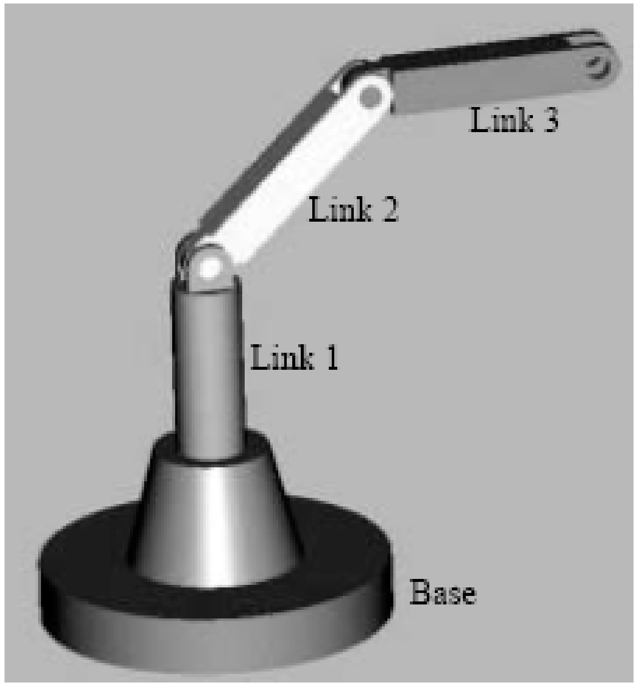


Kinematics – Terminology

- Link is the rigid part of the robot body (e.g. forearm).
- Joint is a part of the robot body which allows controlled or free relative motion of two links (connection element).
- End effector is the link of the manipulator which is used to hold the tools (gripper, spray gun, welding gun . . .).
- Base is the link of the manipulator, which is usually connected to the ground and is directly connected to the world coordinate system.
- Kinematic pair is a pair of links, which relative motion is bounded by the joint connecting them (e.g. base and shoulder connected by J1 axis).



Open chain manipulator kinematics

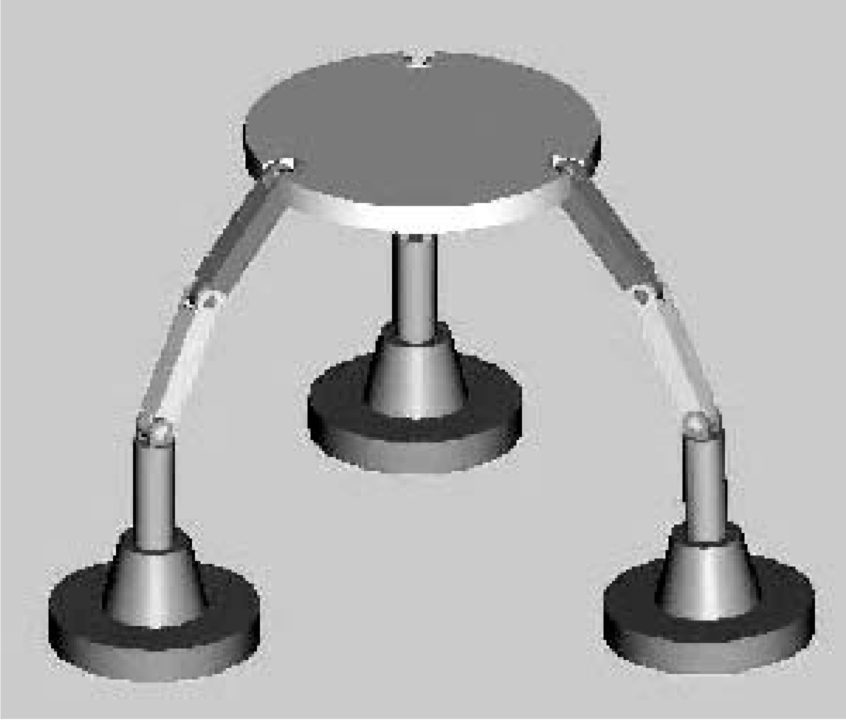


- Mechanics of a manipulator can be represented as a kinematic chain of rigid bodies (links) connected by revolute or prismatic joints.
- Kinematics can be represented by an acyclic graph (tree). Example: human hand.
- One end of the chain is constrained to a base, while an end effector is mounted to the free end of the chain.
- The resulting motion is obtained by composition of the elementary motions of each link with respect to the previous one.

Open chain manipulators, examples

Closed kinematic chain

- Much more difficult. Can be represented by a (general, cyclic) graph.
- Even analysis has to take into account statics, constraints from other links, etc.
- Synthesis of closed kinematic mechanisms is very difficult.
- Main advantage = higher stiffness.



Closed kinematic chain examples

Hybrid chain

Parallel chain

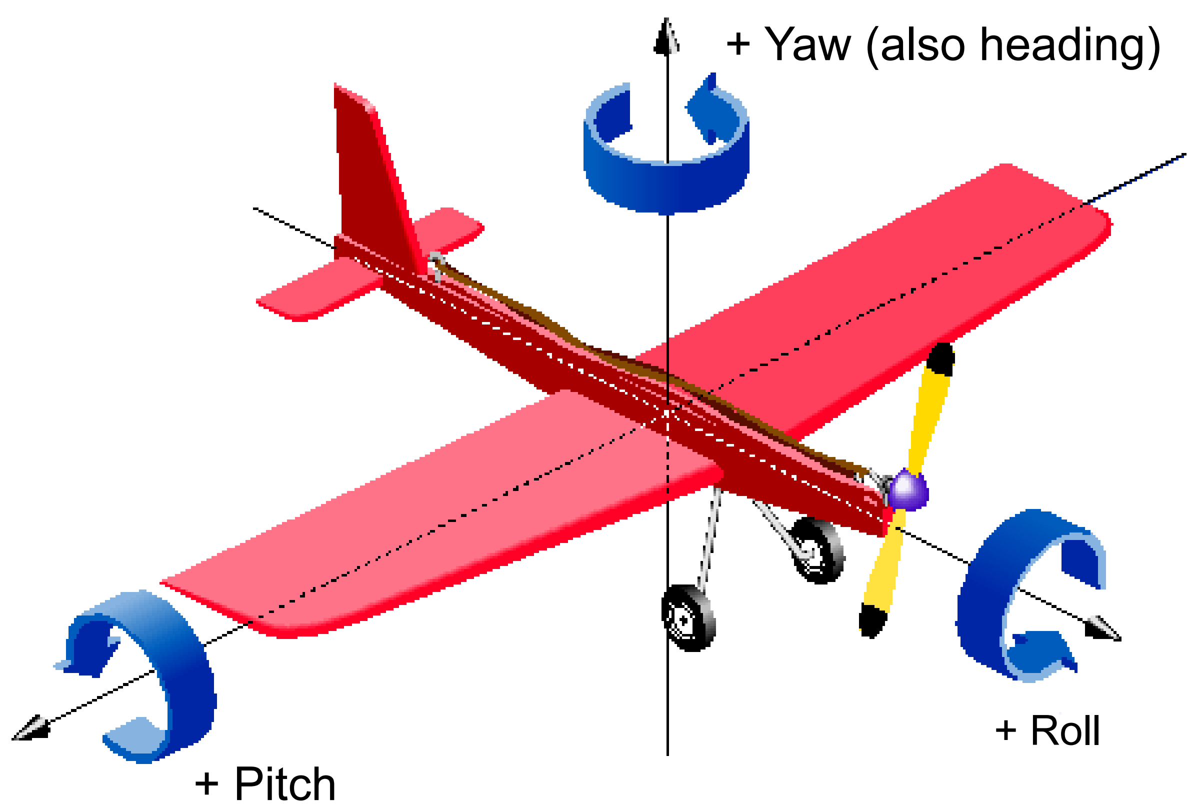
Kinematics vs. differential kinematics

in a special case of an open kinematic chain mechanism, e.g. a robotic manipulator

- Kinematics describes the analytical relationship between the joint positions and the end-effector position and orientation.
- Differential kinematics describes the analytical relationship between the joint motion and the end-effector motion in terms of velocities.

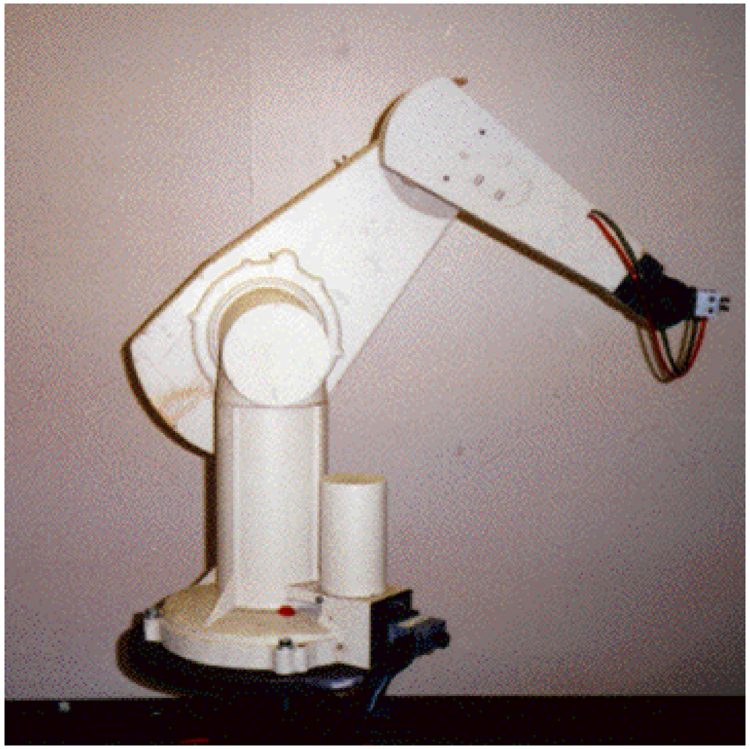
Degrees of Freedem, a free rigid object

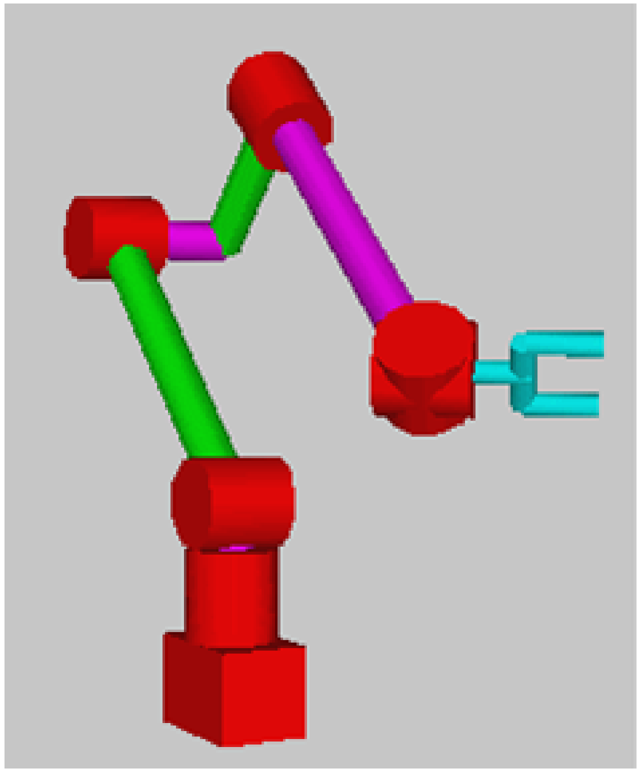
- Q: How many parameters (Degrees of Freedom, DoF) are needed to specify a flying rigid body?
  A: Six, three coordinates of the position x, y, z, and three rotation angles.
- Example: Kinematics of the airplane allows it to move anywhere in the 3D space.



Degrees of freedom, example, question

- Q: How many degrees of freedom (DoF) this manipulator has?

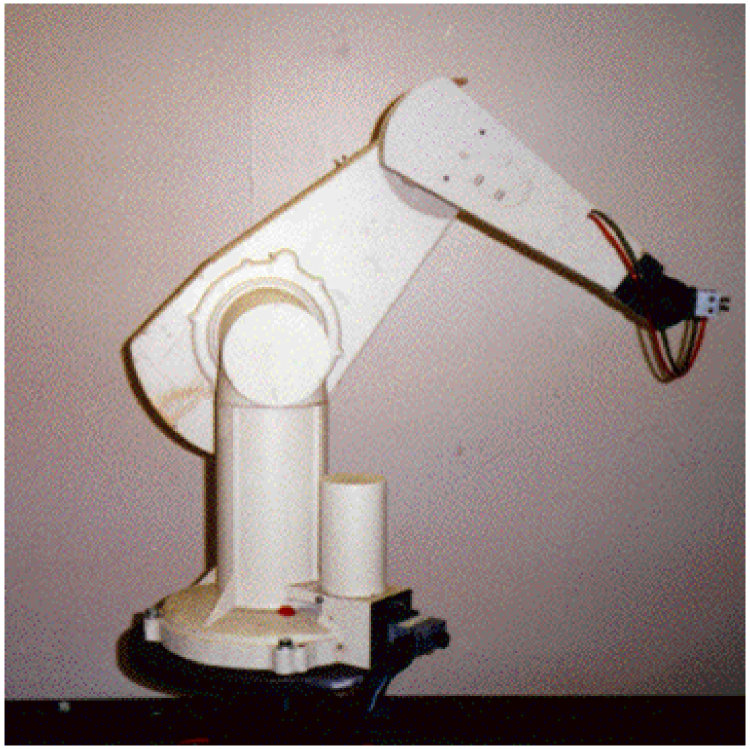


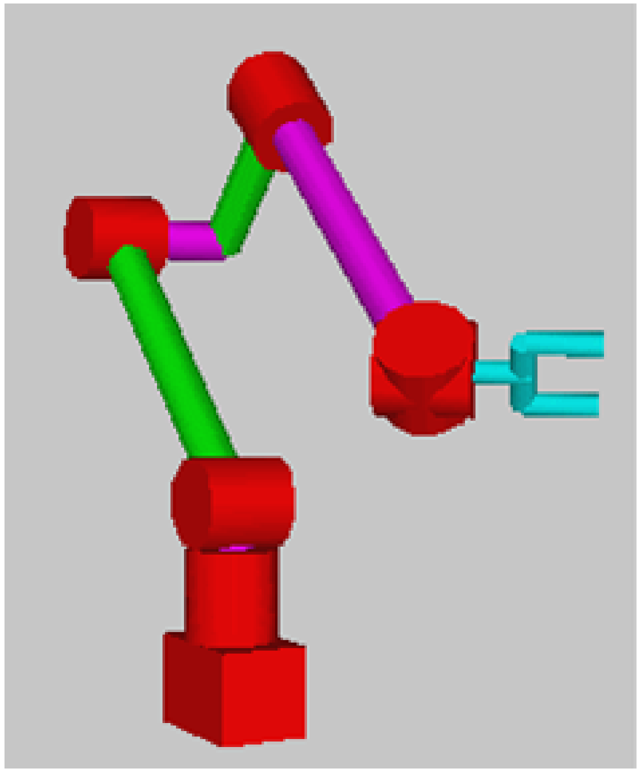


- A:

Degrees of freedom, example, answer

- Q: How many degrees of freedom (DoF) has this manipulator?

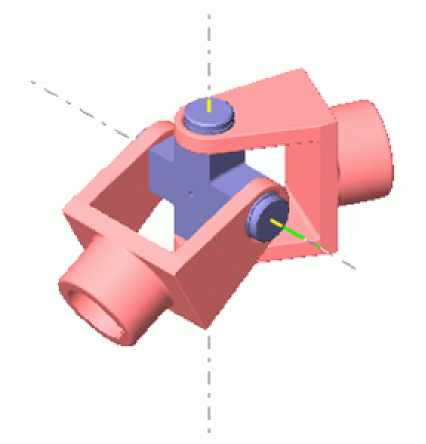




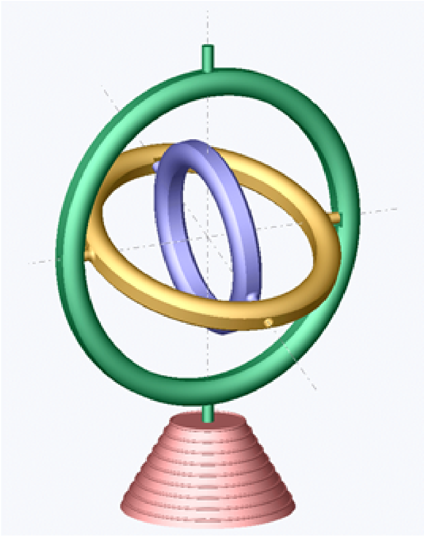
- A: Six again. 2 base + 1 shoulder + 1 elbow + 2 wrist = 6.

Kinematic joints, a quiz

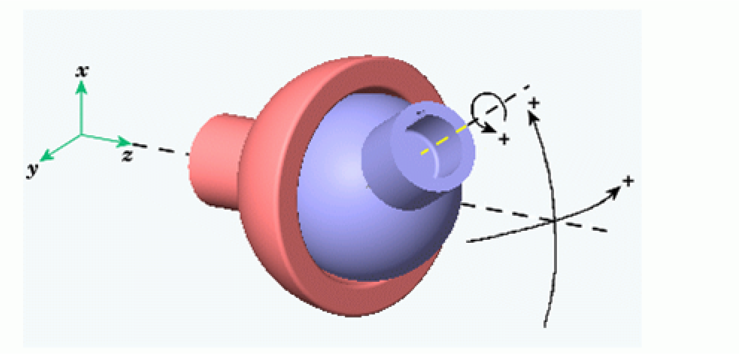
Q: Joints examples: How many degrees of freedom they have?



Cardan joint



3D gimbal

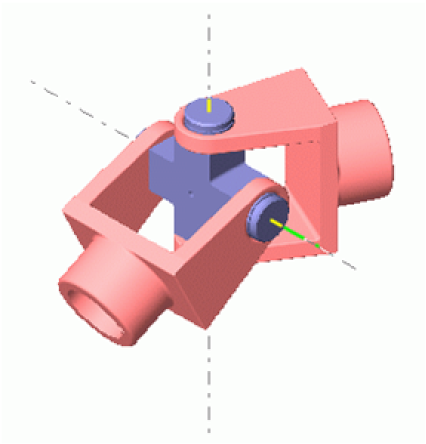


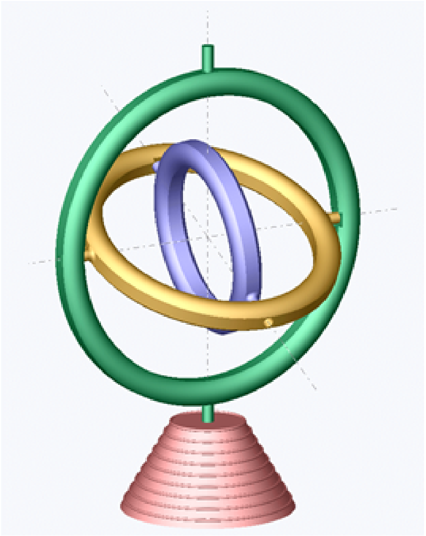
spherical

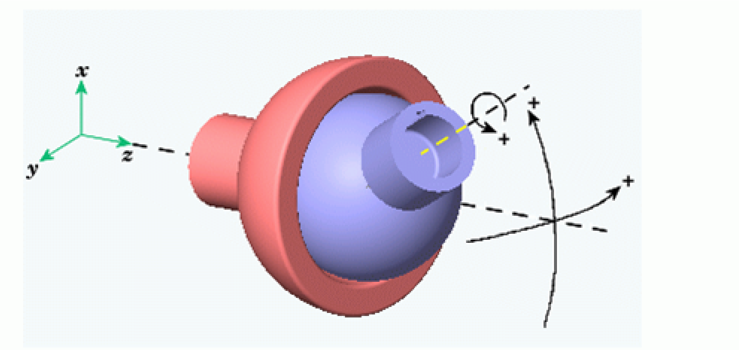
A:

Kinematic joints, answers to the quiz

Q: How many degrees of freedom?

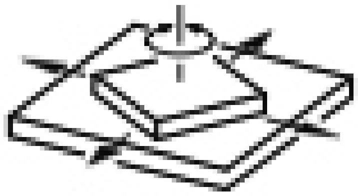






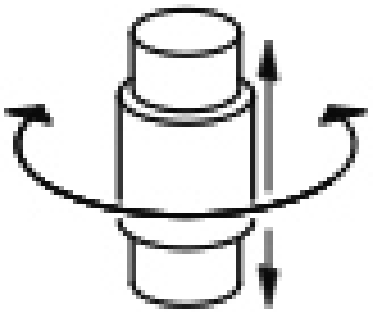
	Cardan joint	3D gimbal	spherical
A:	2 DOFs	3 DOFs singularities	3 DOFs no singularities

Kinematic joints

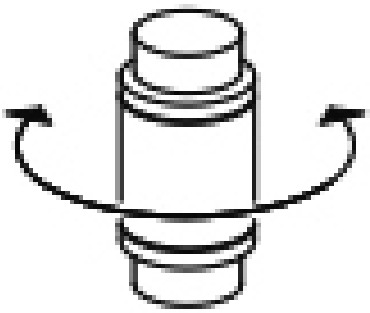


Planar
3 DOF

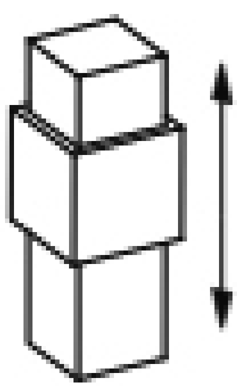
Spherical
3 DOF



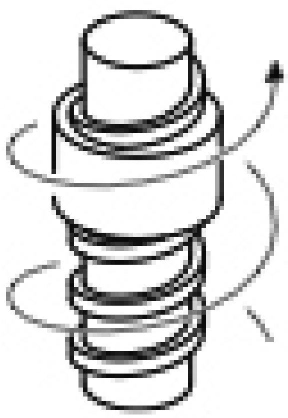
Cylindrical
2 DOF



Revolute
1 DOF

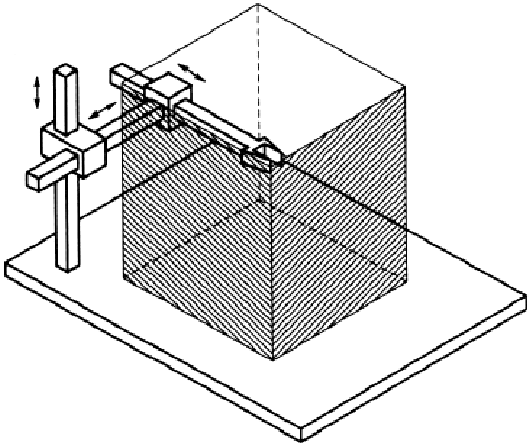


Prismatic
1 DOF

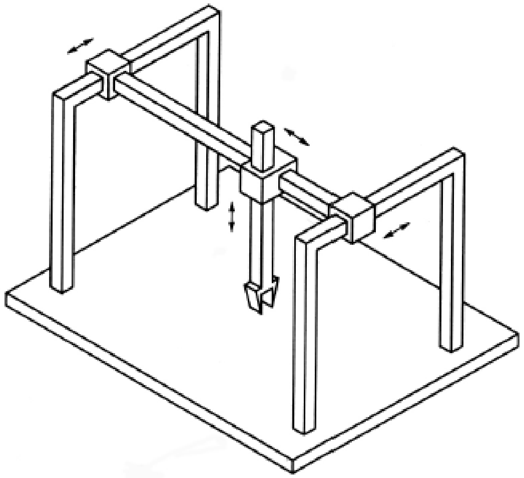


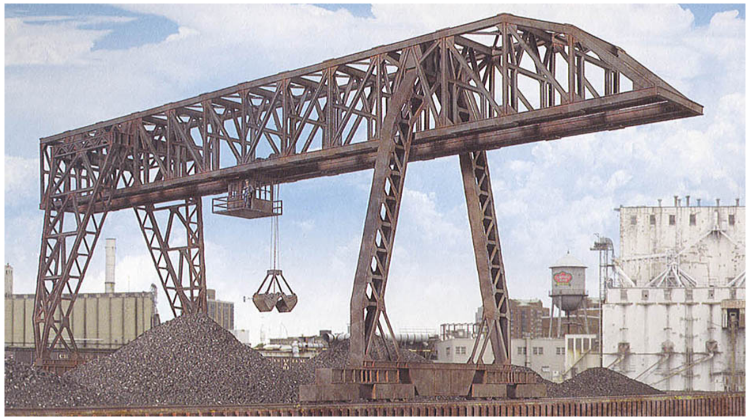
Helical
1 DOF

Structure of manipulators – Cartesian – PPP



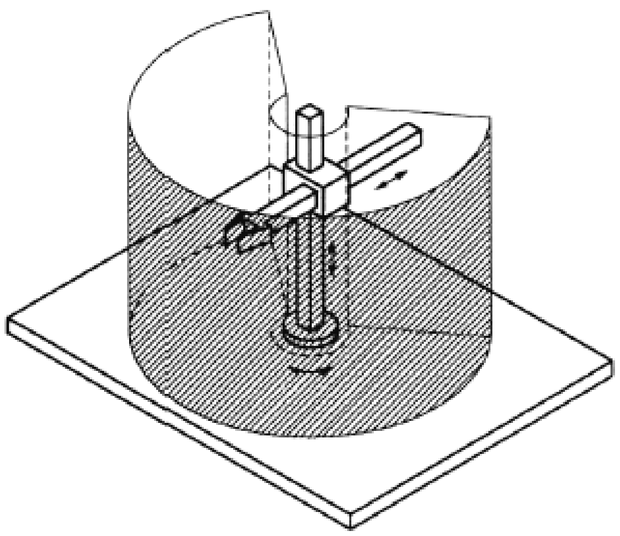
Cartesian



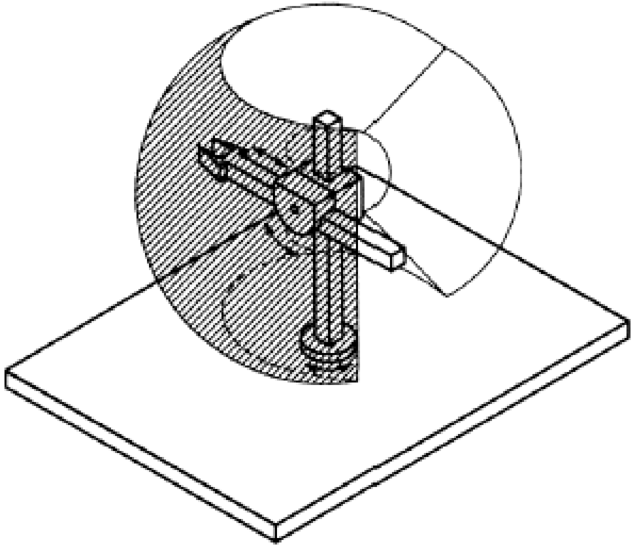


Gantry

Structure of manipulators – Cylindrical – RPP

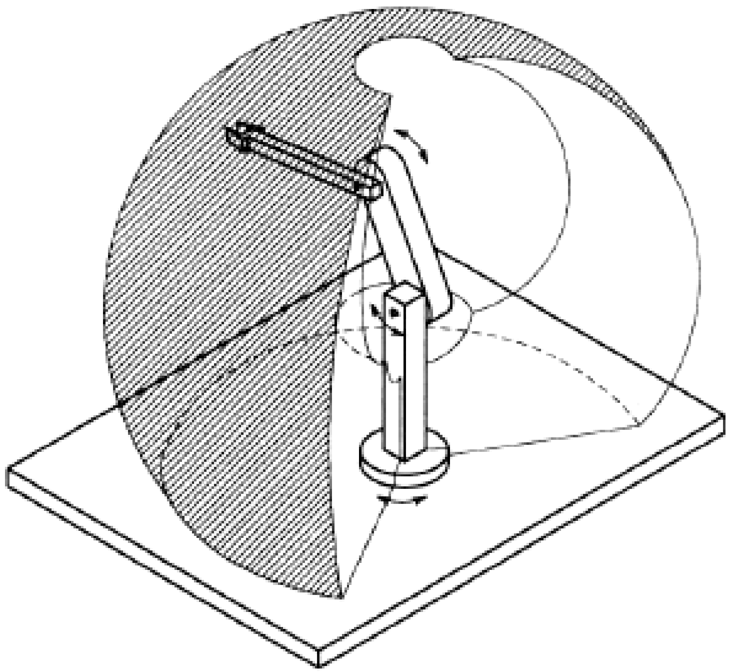


Structure of manipulators – Spherical – RRP

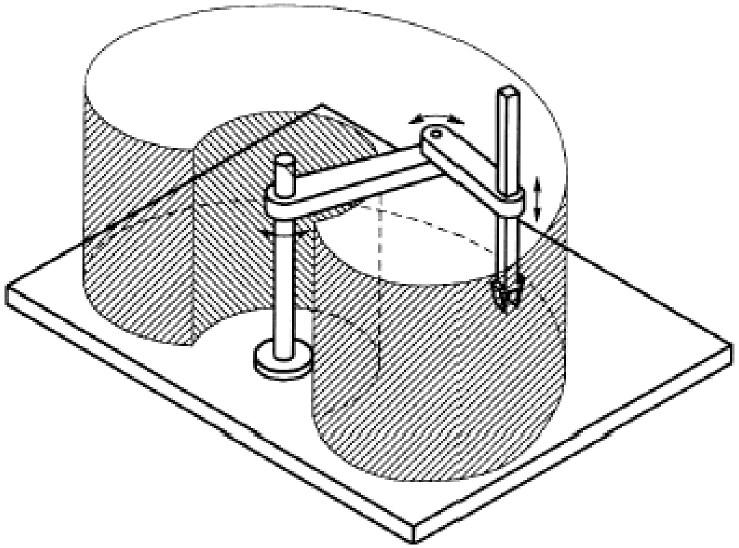


Structure of manipulators – Angular – RRR

Called also: anthropomorphic



Structure of manipulators – SCARA – RRRP

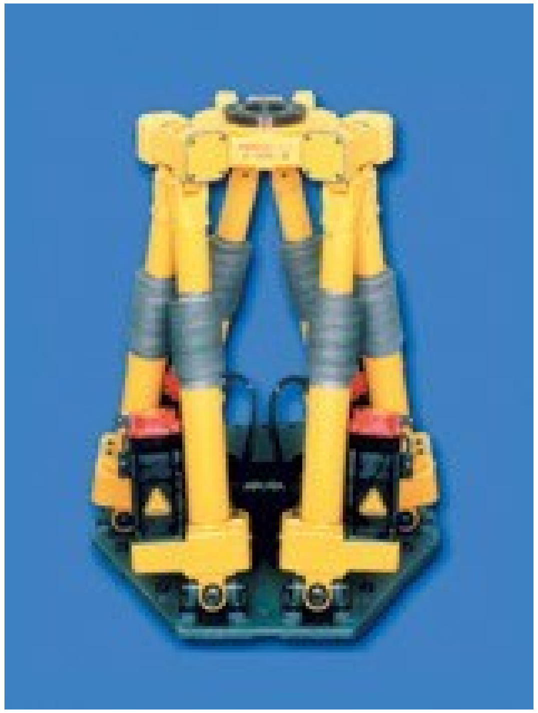


Structure of manipulators – Stewart platform

- Parallel kinematics.
- 6 DoFs.
- 6 prismatic actuators, commonly hydraulic jacks.
- Called also 6-axes platform or hexapod.
- Designed by V. E. Gough in 1954 for tyre testing.
- Published by D. Stewart in 1965.

Stewart platform, applications

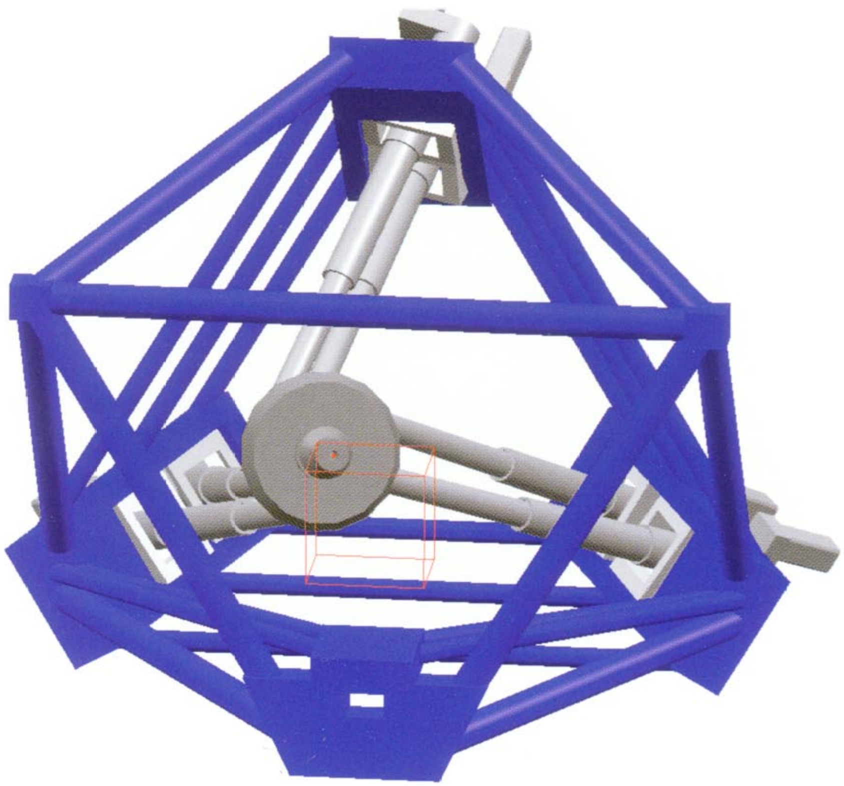
Large jacks

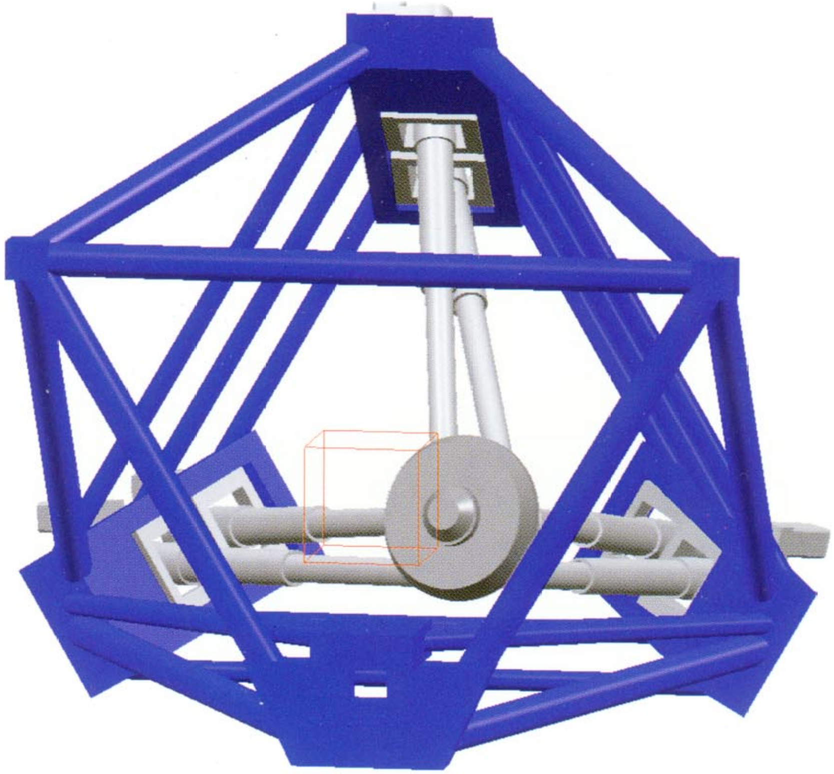


FANUC

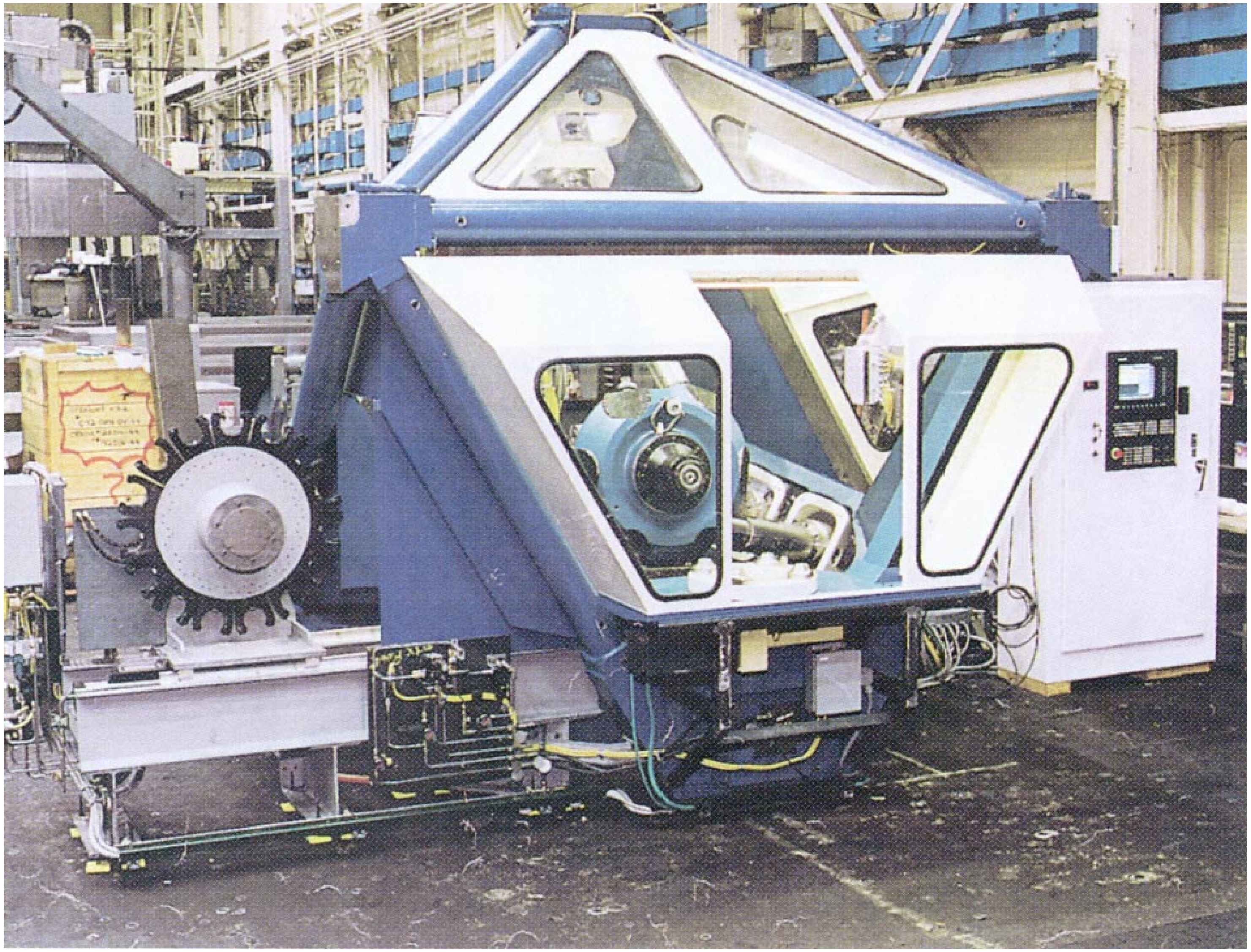
Flight simulator

Hexamod



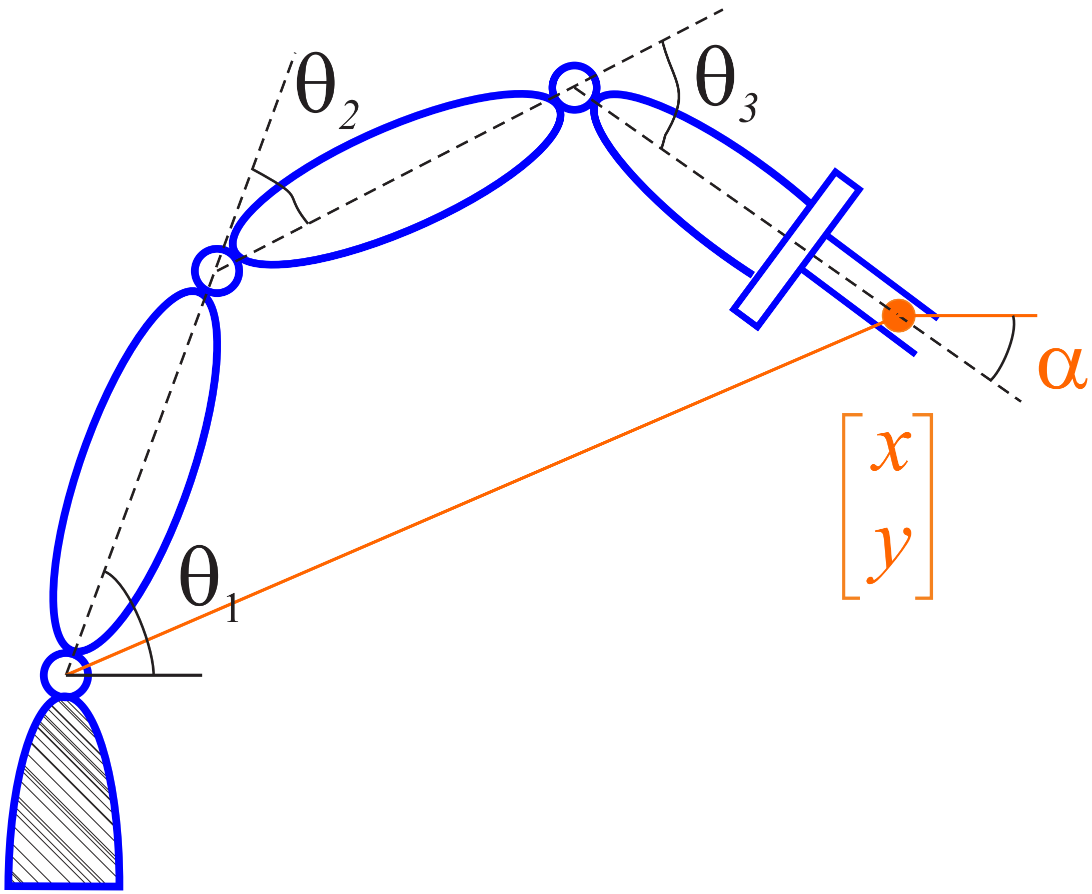


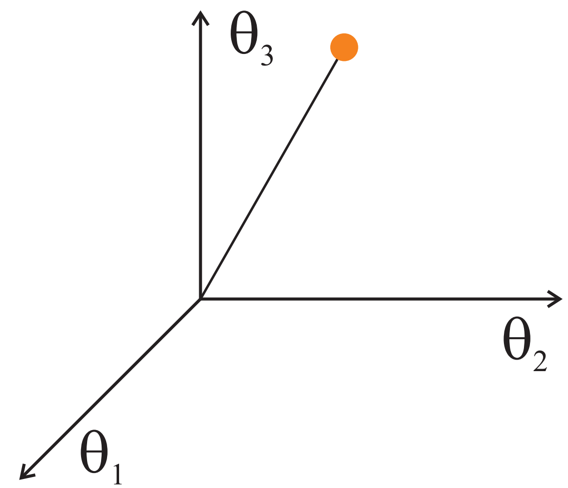
Real hexamod



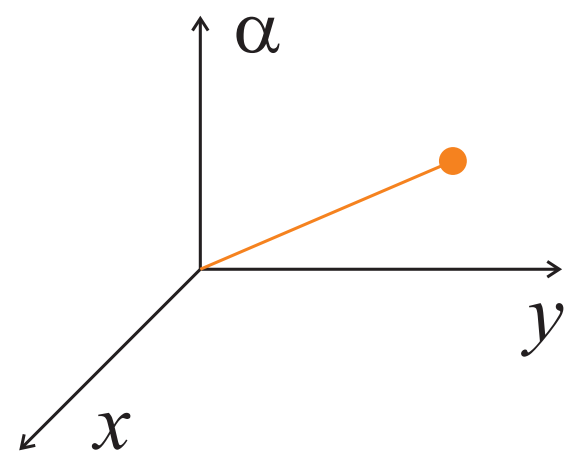
Joint and operational spaces, motivation

Example: a 3 DOF planar manipulator





Joint space.



Operational space.

Concepts:

joint space, joint coordinates;

operational space, operational coordinates

Direct vs. inverse kinematics

In an open chain kinematic manipulator robotics, there are two kinematic tasks:

1. Direct (also forward) kinematics

   Given: Joint relations (rotations, translations) for the robot arm.

   Task: What is the orientation and position of the end effector?

2. Inverse kinematics

   Given: The desired end effector position and orientation.

   Task: What are the joint rotations and orientations to achieve this?

In a more general case of close kinematic chain mechanisms, a more general statement is needed:

1. Direct kinematics

   Given: the geometric structure of the manipulator and the values of a number of joint positions equal to the number of degrees of freedom of the mechanism.

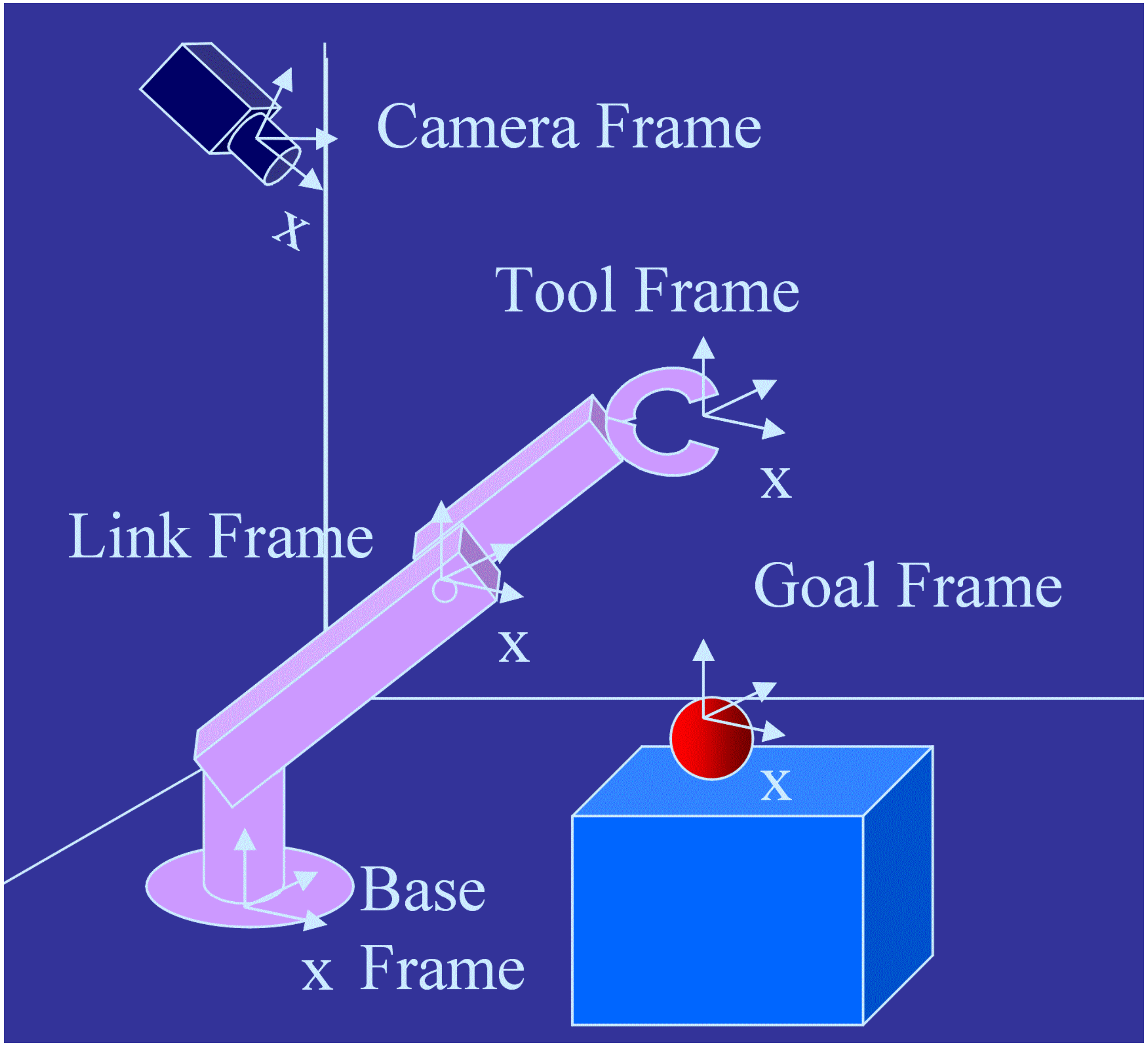
   Task: Find a relative position and orientation of any two designed joints.

2. Inverse kinematics

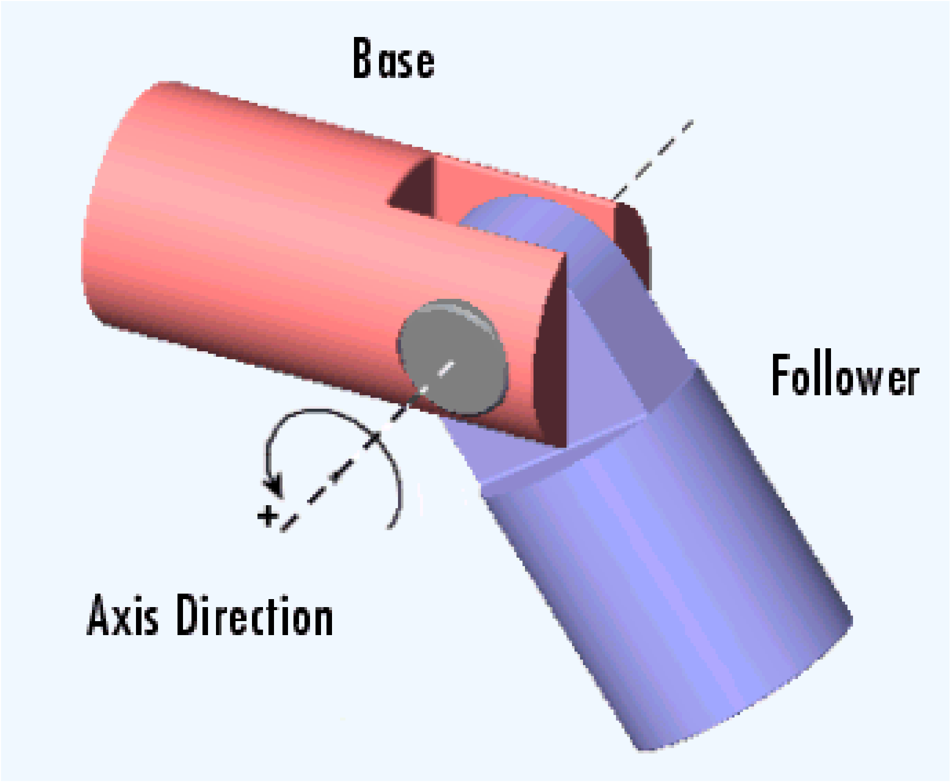
   Given: a relative position and orientation of any two designed joints.

   Task: Find values of all joints position and orientations.

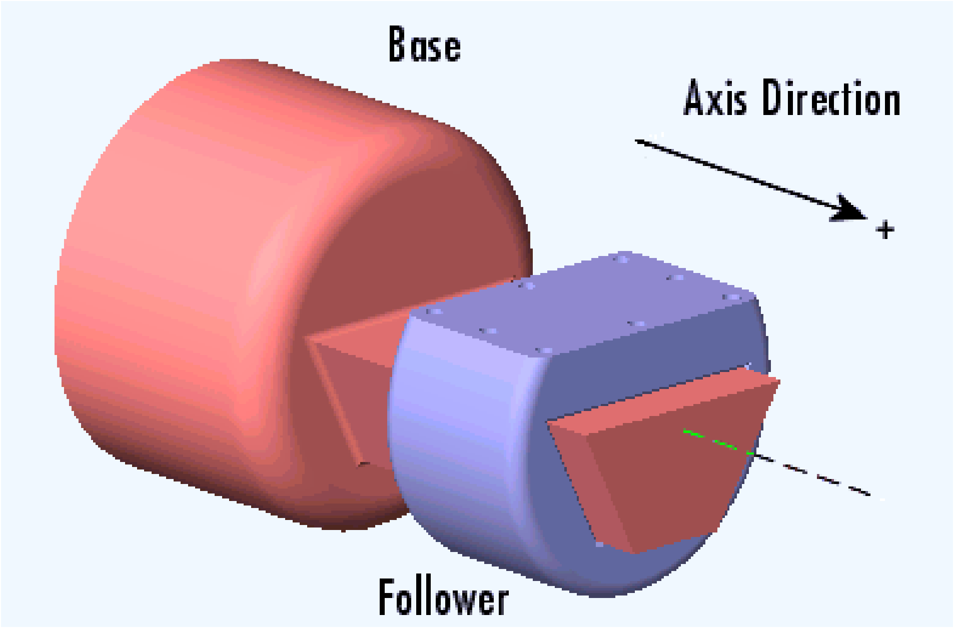
Coordinate frames



Two basic types of joints

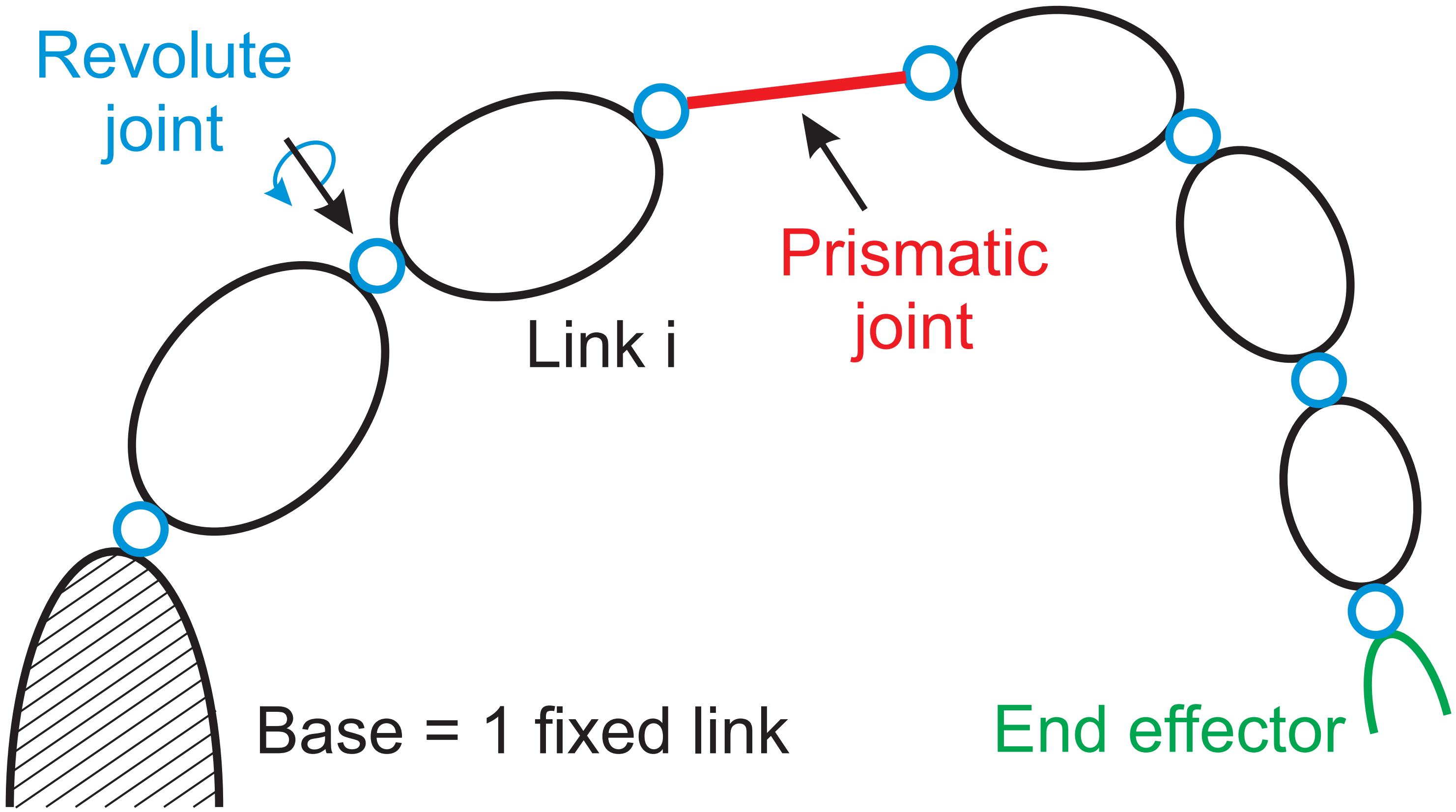


Revolute



Prismatic

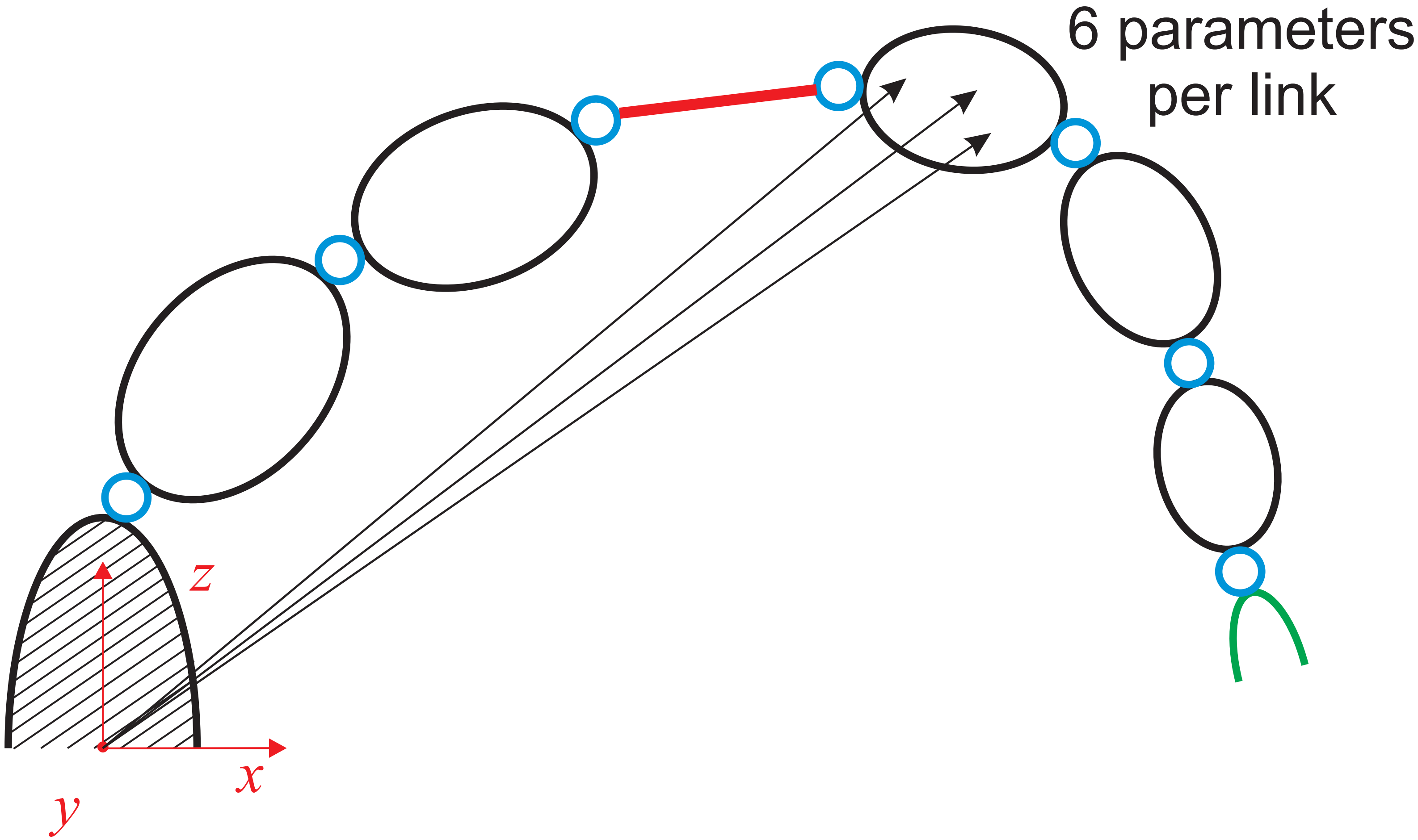
Manipulator description



- Links: n moving links, 1 fixed link (base).
- Joints: revolute (1 DOF), prismatic (1 DOF).

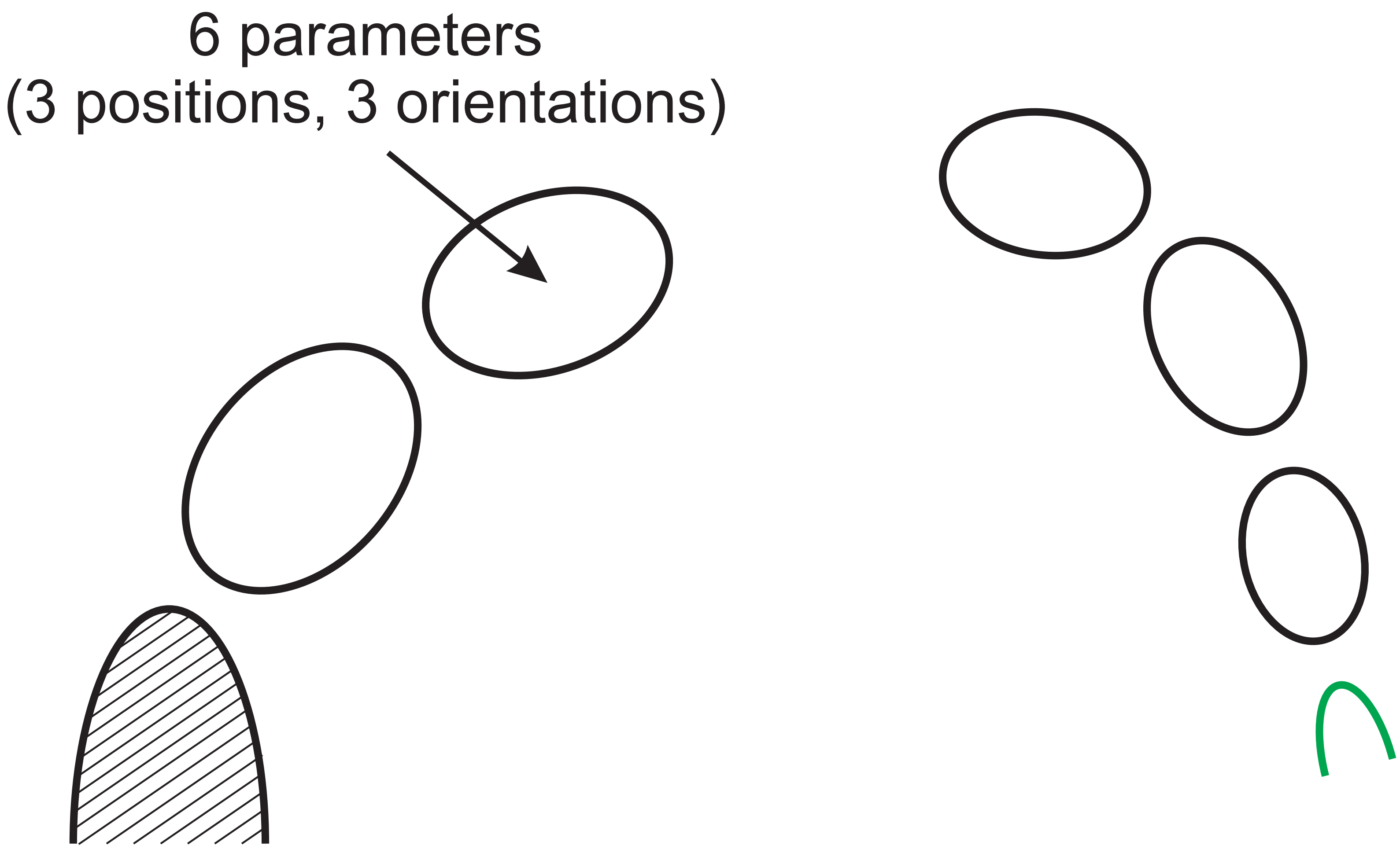
Configuration parameters

- Configuration parameters are given by a set of positions describing the full configuration of the system.



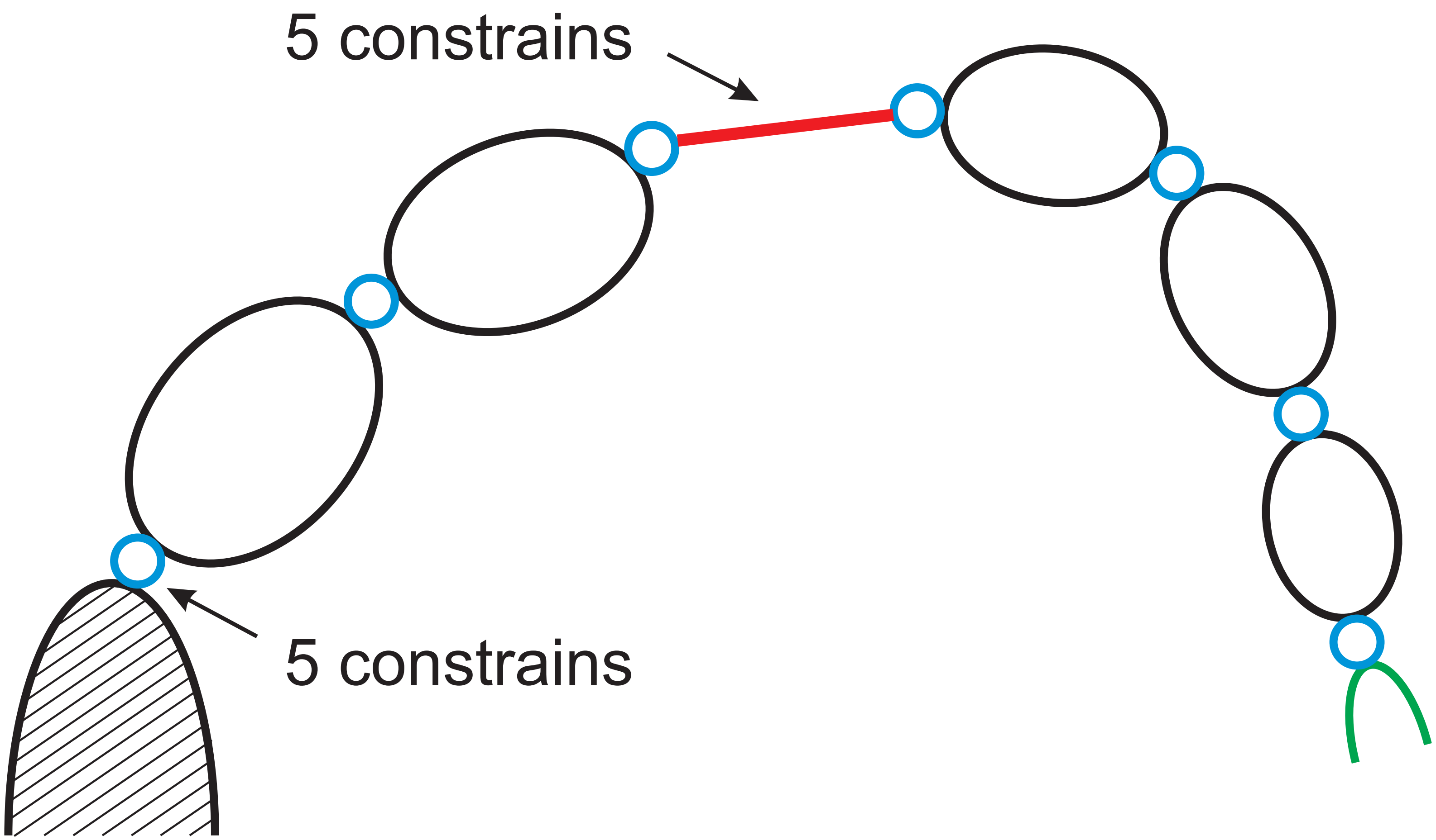
- Generalized coordinates – a set of independent configuration parameters.
- Degrees of freedom – number of generalized coordinates.

Generalized coordinates (1)



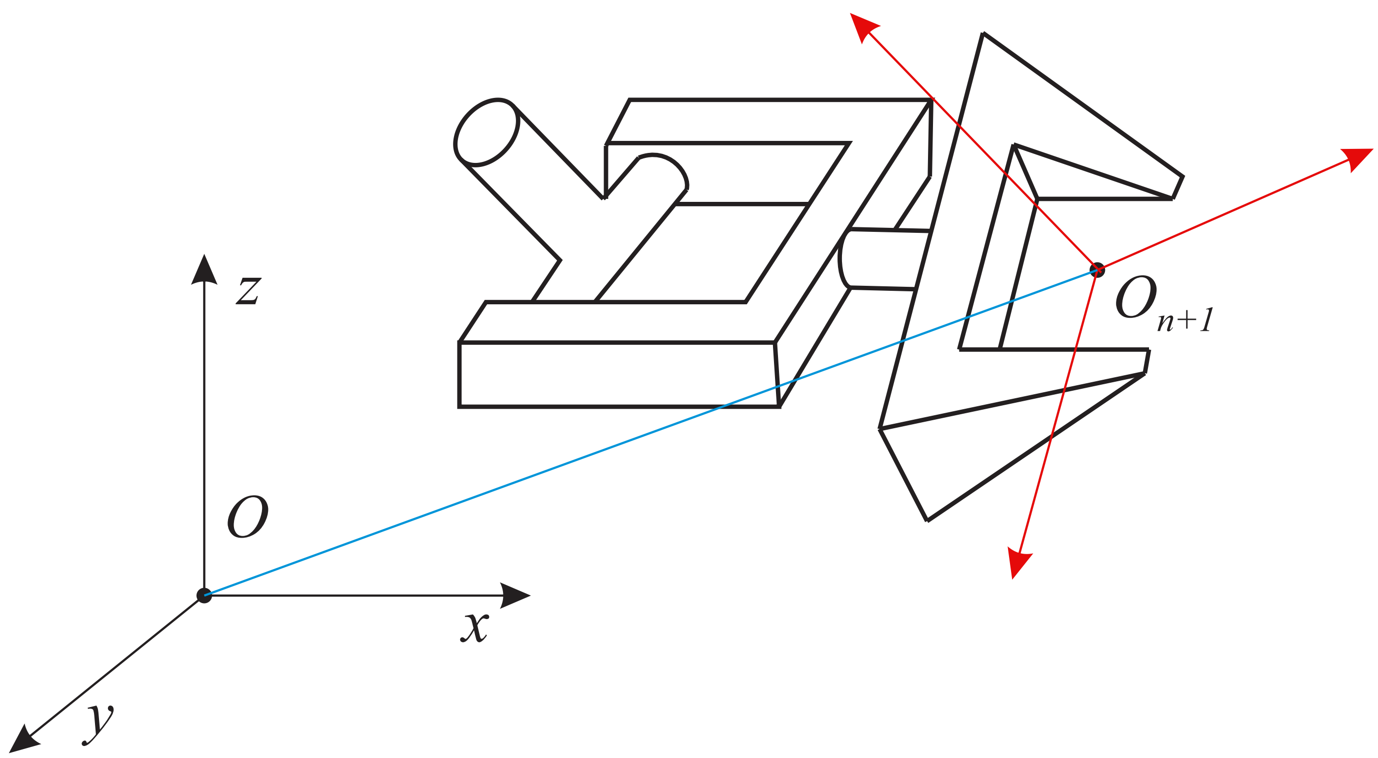
n unconstraint moving links $\Rightarrow$ $6n$ parameters.

Generalized coordinates (2)



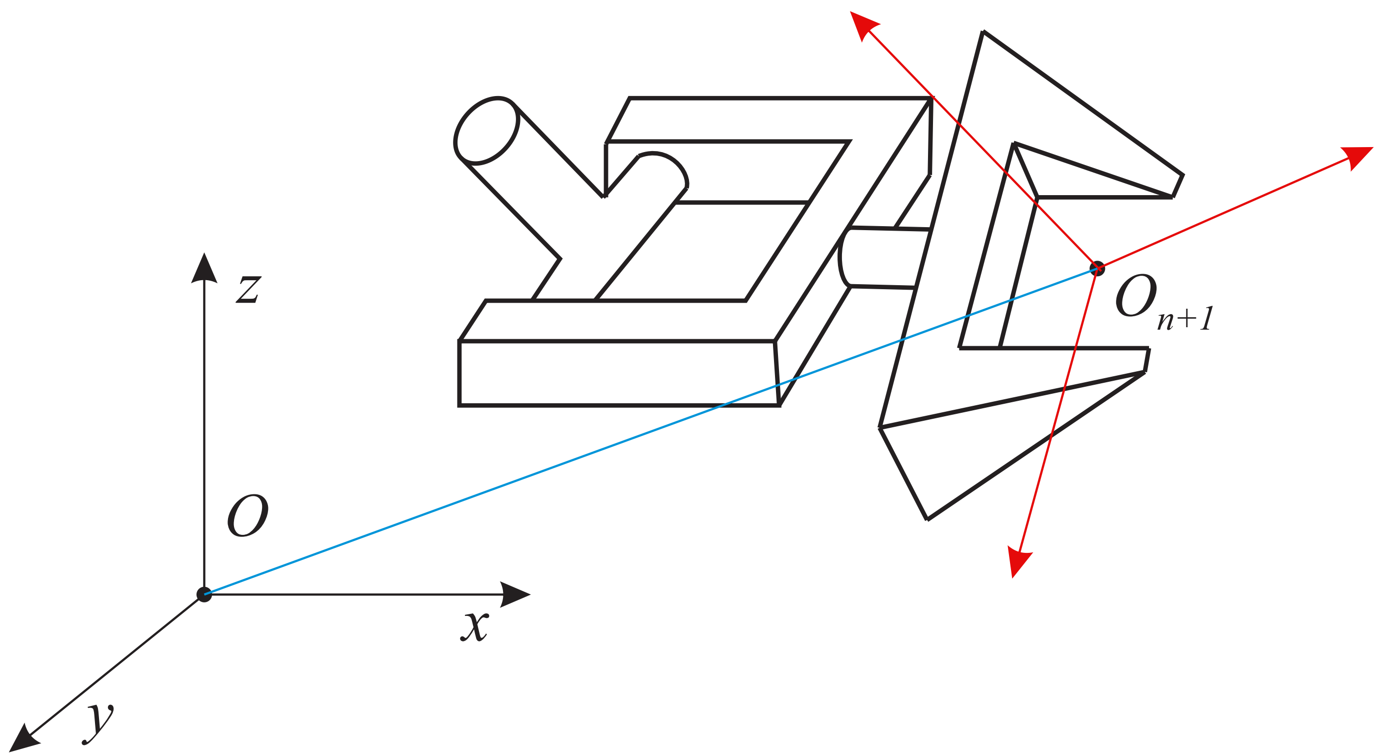
- n moving links $\Rightarrow$ $6n$ parameters.
- n 1 DOF joints $\Rightarrow$ $5n$ constraints.
- The system has $6n - 5n = n$ DOFs.

End-effector configuration parameters



- O – Origin of the world coordinates at the manipulator base.
- O_{n+1} – Operational point, the representative point of the end-effector.
- $(x_1, x_2, \ldots, x_m)$ – A set of parameters, which specifies the end-effector position and orientation with respect to coordinate system O.

Operational (joints) coordinates

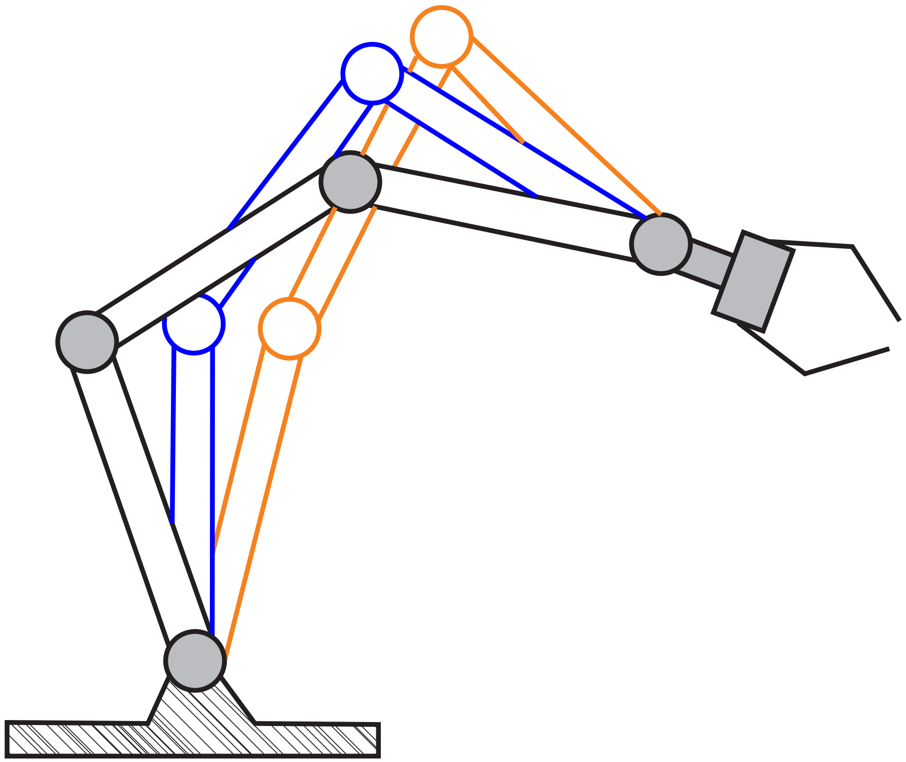


- $(x_1, x_2, \ldots, k)$ – A set of k, $k \leq m$ independent configuration parameters.
- m_0 – number of end-effector degrees of freedom.

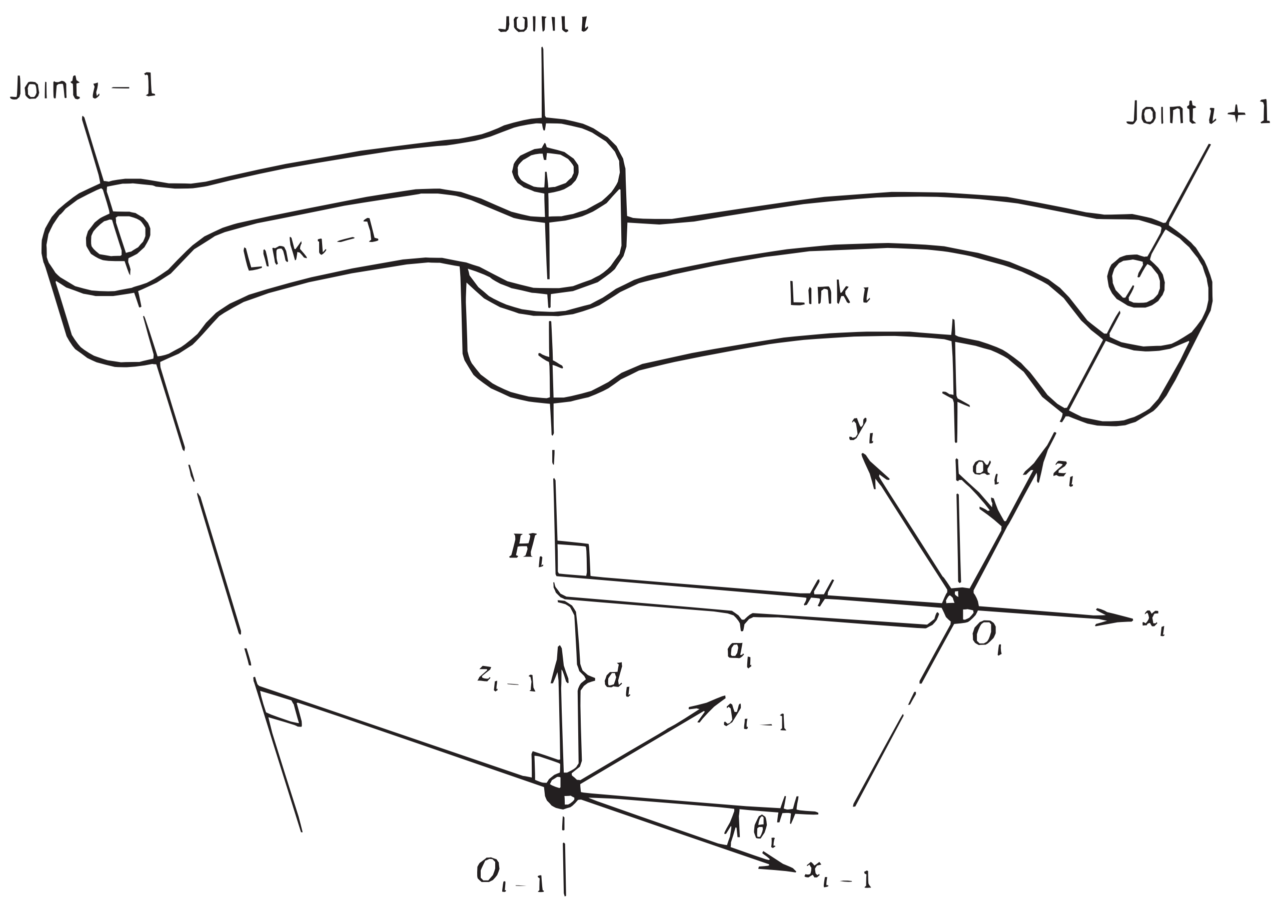
Manipulator redundancy

- n is the degree of freedom of a manipulator (robot).
- m_0 is the number of the end effector DoFs, 3 in the example.
- A manipulator (robot) is redundant if $n > m_0$.
- Degrees of redundancy $= n - m_0$.

Example: a planar manipulator in 2D.

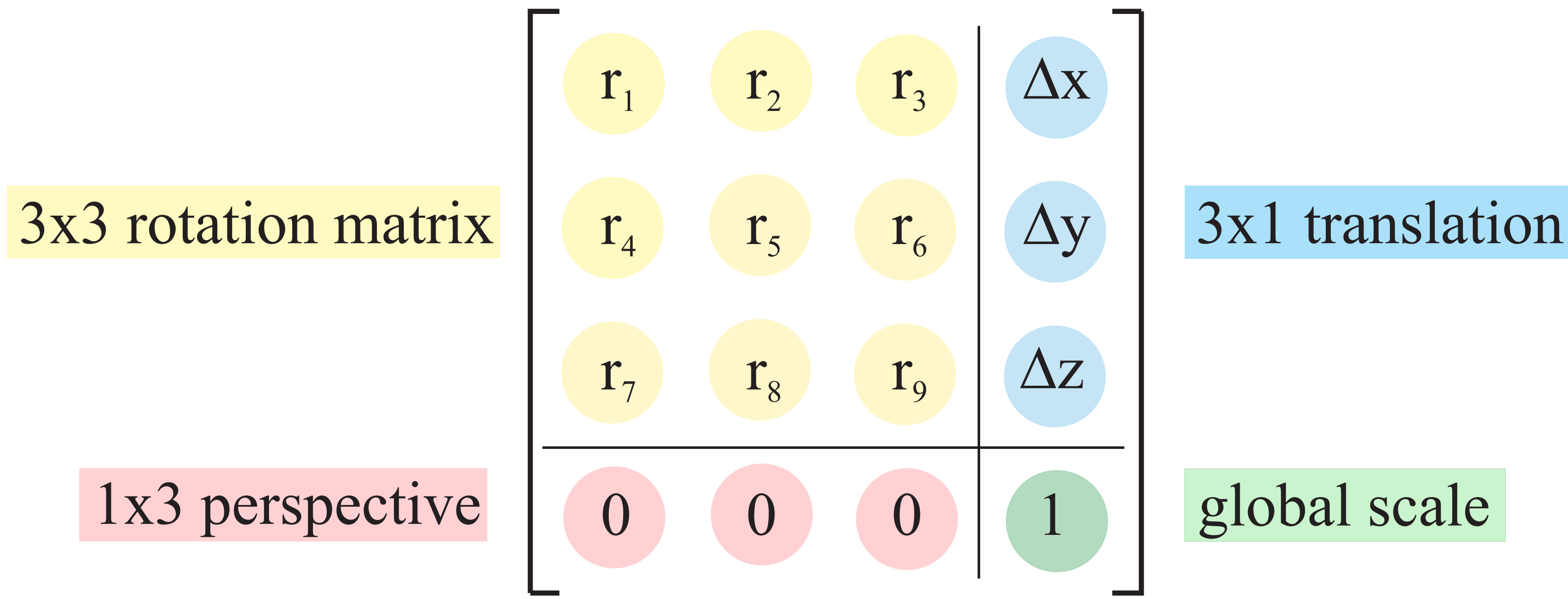


Two frames kinematic relationship



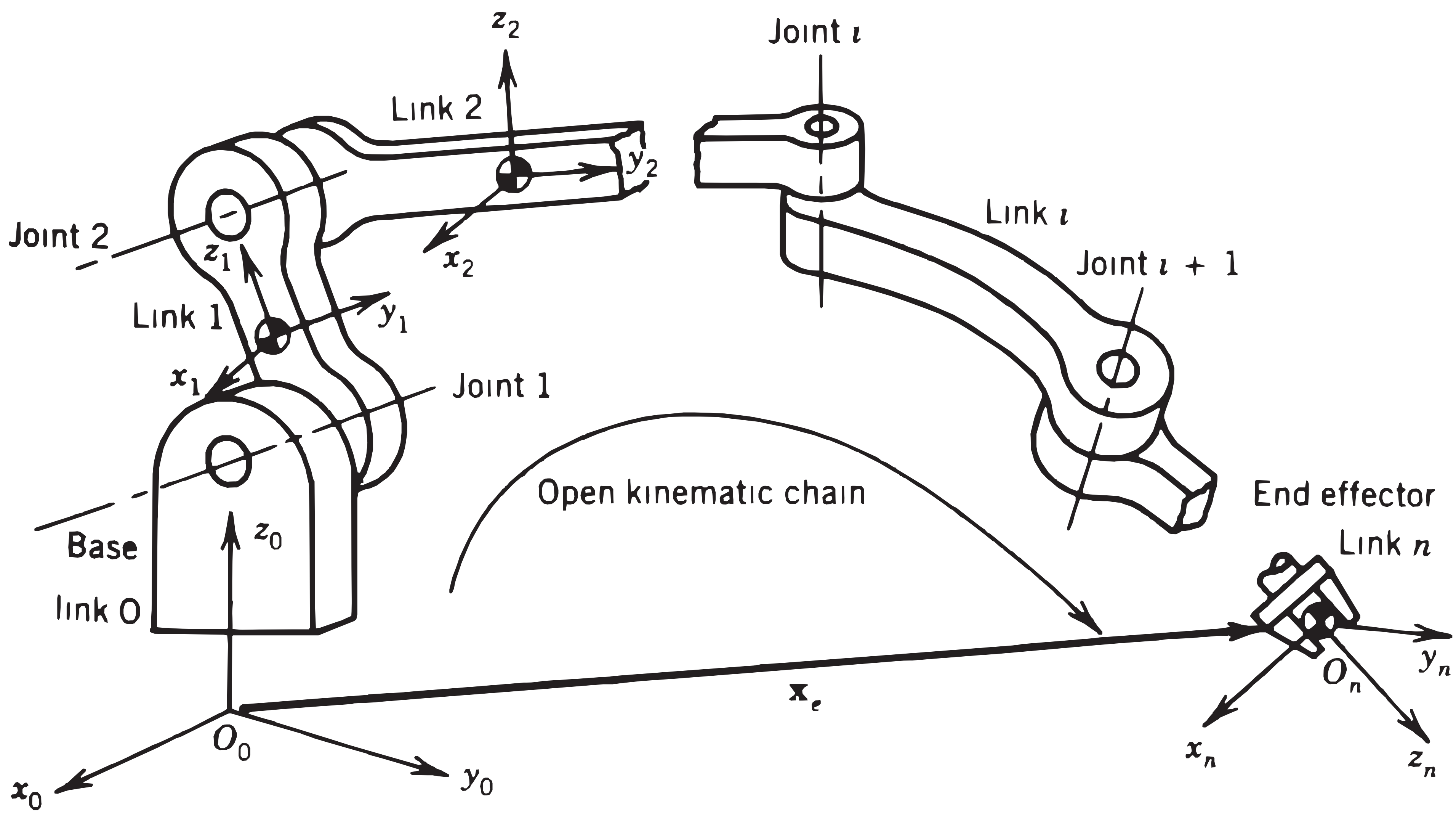
- There is a kinematic relationship between two frames, basically a translation and a rotation.
- This relationship is represented by a 4 $\times$ 4 homogeneous transformation matrix.

Homogeneous transformation



Rotation matrix R is orthogonal $\Leftrightarrow R^\top R = I \Rightarrow$ 3 independent entries, e.g., Euler angles.

Kinematic open chain



Direct vs. inverse kinematics, a reminder

In an open chain kinematic manipulator robotics, there are two kinematic tasks:

1. Direct (also forward) kinematics

   Given: Joint relations (rotations, translations) for the robot arm.

   Task: What is the orientation and position of the end effector?

2. Inverse kinematics

   Given: The desired end effector position and orientation.

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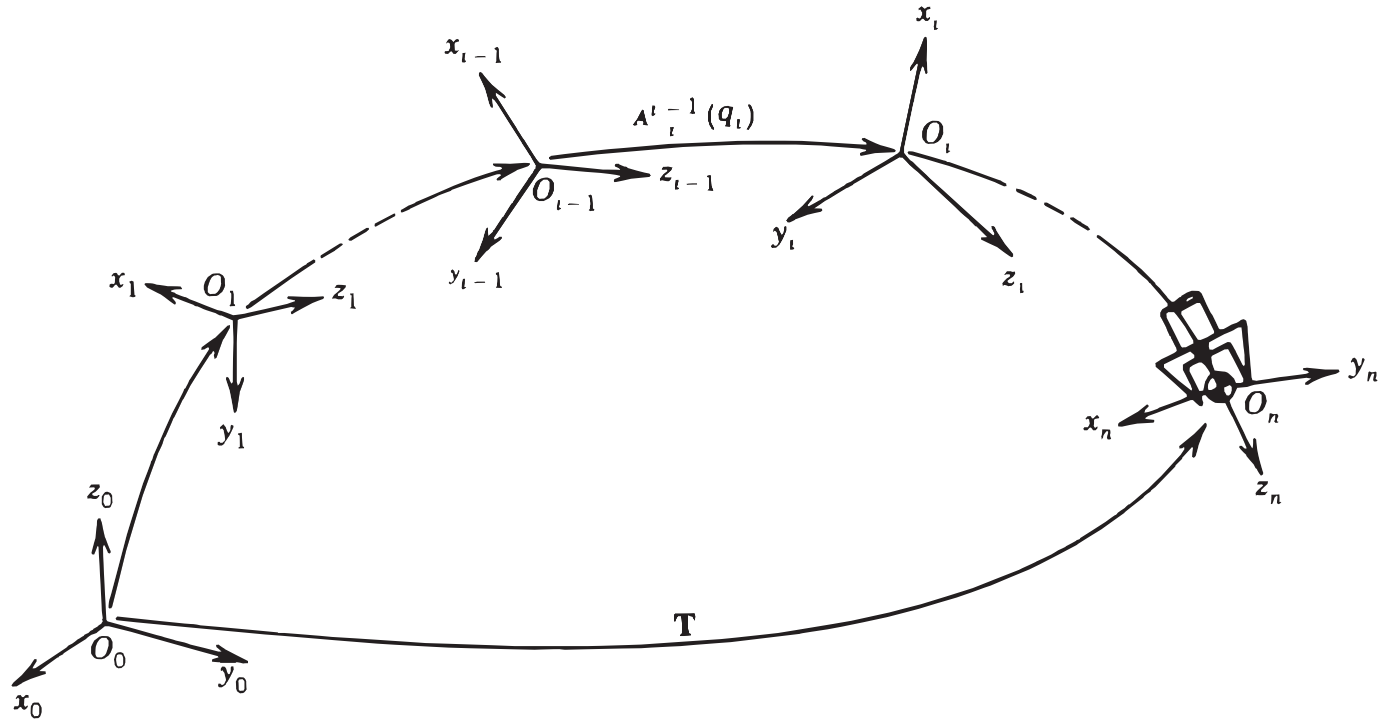
Direct kinematics



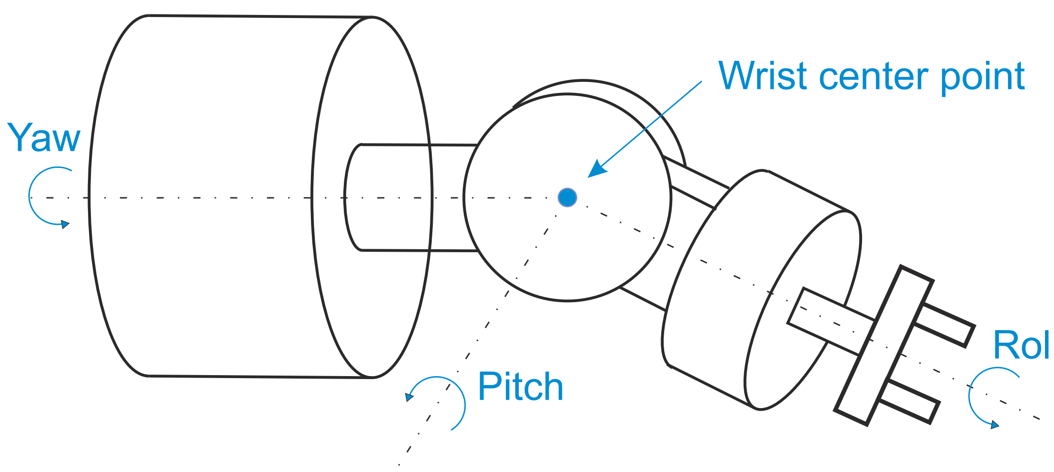
- One joint: $\mathbf{x}_i = A\mathbf{x}_{i-1}$.
- Chain of joints: $\mathbf{x}_{n-1} = A_{n-1}\, A_{n-2}\, \ldots\, A_1\, A_0\, \mathbf{x}_0$.
- Easy to compute (matrix multiplication).
- Unique solution.

Inverse kinematics

- For an open chain kinematic mechanism (a robot), the inverse kinematic problem is difficult to solve.
- The robot controller must solve a set of non-linear simultaneous algebraic equations.
- Source of problems:
  - Non-linear equations ($\sin$, $\cos$ in rotation matrices).
  - The existence of multiple solutions.
  - The possible non-existence of a solution.
  - Singularities.

Kinematic decoupling ⇒ inverse kinematics becomes simpler

- Divide and conquer strategy. Decouple the problem into independent subproblems.
- General inverse kinematic (IK) task is difficult. However, for 6-DOF manipulators with the last 3 joint axes intersecting at one point, IK simplifies to two simpler tasks: (a) inverse position kinematics, (b) inverse orientation kinematics.
- The spherical wrist. Positioning of the wrist + positioning within the wrist.



- Design conventions, e.g. Denavit-Hartenberg systematic frame assignment.

Methods solving the inverse kinematics task

1. Closed-form solutions. Relevant for industrial manipulators.
   - Algebraic methods.
   - Geometric methods.
2. Numerical methods.
   - *Symbolic elimination methods*: involve analytical manipulations to eliminate variables from a system of nonlinear equations to reduce it to a smaller set of equations.
   - *Continuation methods*: involve tracking a solution path from a start system with known solutions to a target system.
   - *Iterative methods*: are in general based on Newton-Raphson method for finding roots using 1st order approximation of the original algebraic equation. They converge in a single solution (from several possible) based on the initial guess.