Fair and Adventurous Enumeration of Quantifier Instantiations

Mikoláš Janota[1] Haniel Barbosa[2]
Pascal Fontaine[3] Andrew Reynolds[4]

[1] Czech Technical University in Prague

[2] Universidade Federal de Minas Gerais

[3] University of Liège

[4] University of Iowa

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Motivation — SMT instantiation

Disprove:

$$(\forall x\ f(x) \geq x) \wedge (\forall y\ f(y) \leq 5)$$

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$x \mapsto 6$ $\downarrow$ $\quad$ $\downarrow$ $y \mapsto 6$

$$f(6) \geq 6 \wedge f(6) \leq 5$$

Background: Herbrand

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- Example

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\wedge \quad & (\forall x)(f(x) = x)
\end{aligned}
$$

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Instantiation:

$$\begin{array}{ll} & f(f(c)) \neq c \\ \wedge & f(c) = c \\ \wedge & f(f(c)) = f(c) \end{array}$$

Herbrand universe: $\{f^i(c) \mid c \in \mathbb{N}_0\}$

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  $t_3 \in (G \land \phi[t_1/x] \land \phi[t_2/x])$
- Still infinite but finite in each step!
  [Ge and de Moura, 2009, Reynolds et al., 2018]

Warning!

(Improved) Herbrand does not hold for theories

$$c = 21 \wedge (\forall m, n)((m > 1 \wedge n > 1) \Rightarrow (mn \neq c))$$

SMT Quantifiers by Enumeration

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- Question:
  What order?

Old Age Is Good!

$$G \wedge \forall x.\, \phi$$

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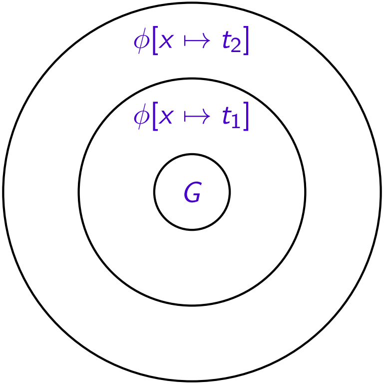

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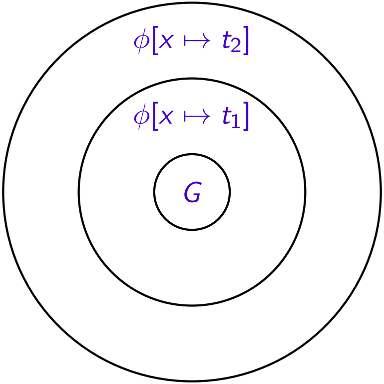


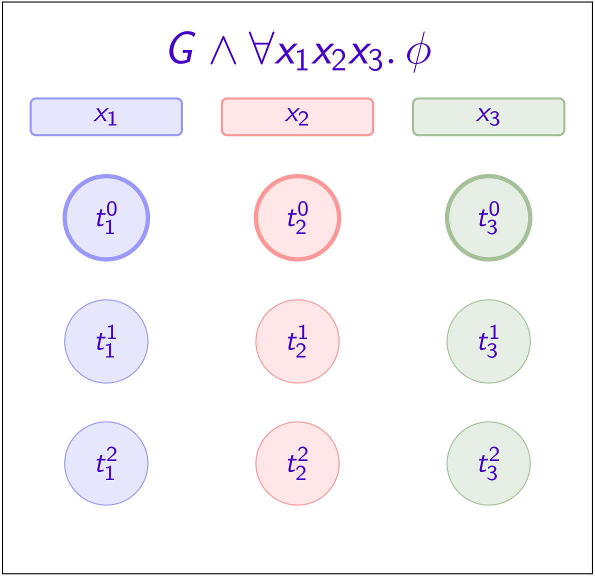
$$G \begin{cases} t_1 \\ t_2 \\ \dots \end{cases}$$

$$\phi[x \mapsto t_1] \begin{cases} \\ \dots \end{cases}$$

$$\phi[x \mapsto t_2] \begin{cases} \\ \dots \end{cases}$$

$$\dots$$

What about Tuples?



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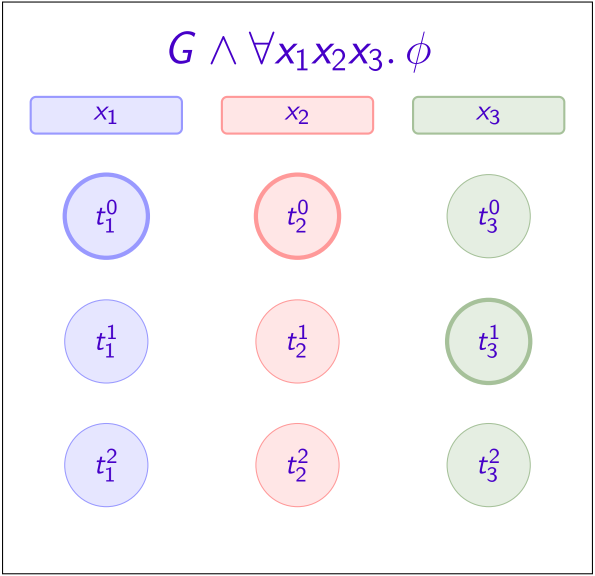

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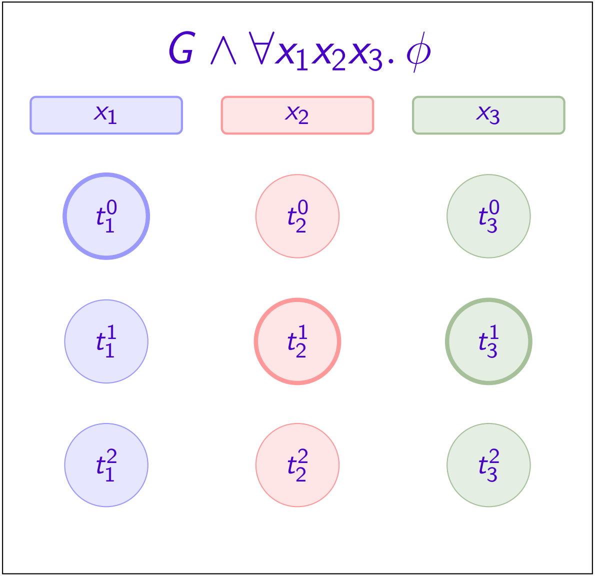

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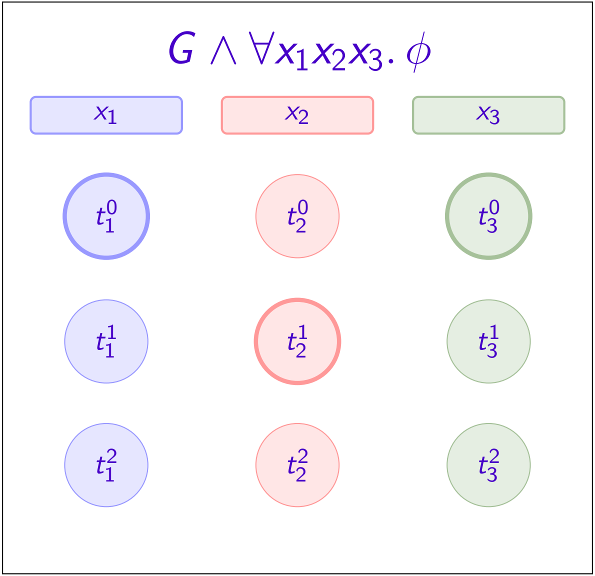
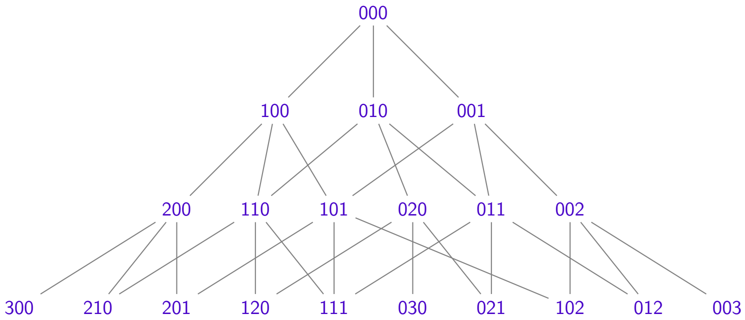


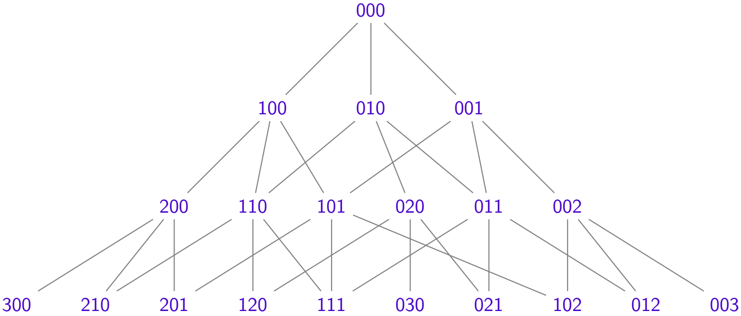
General Idea

1. Pick best term for each variable.



General Idea

1. Pick best term for each variable.
2. Then go worsening as little as possible.



Different Strategies: Maximal Digit

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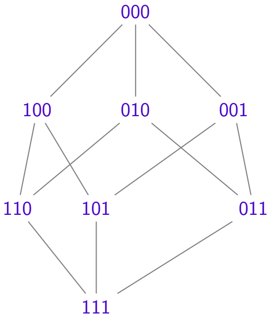

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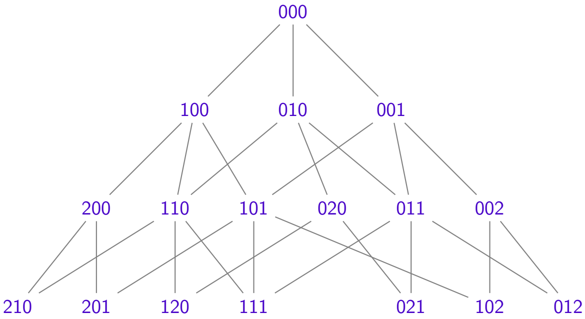

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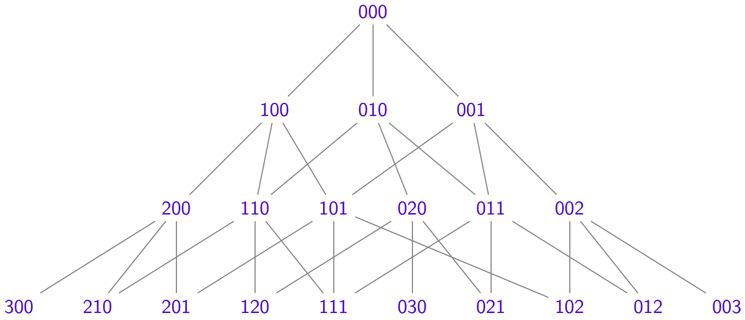

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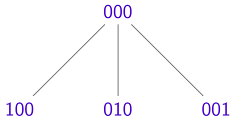

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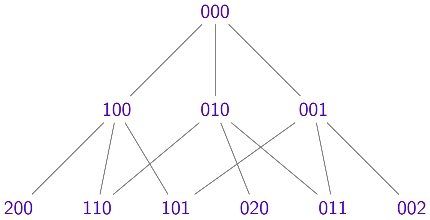

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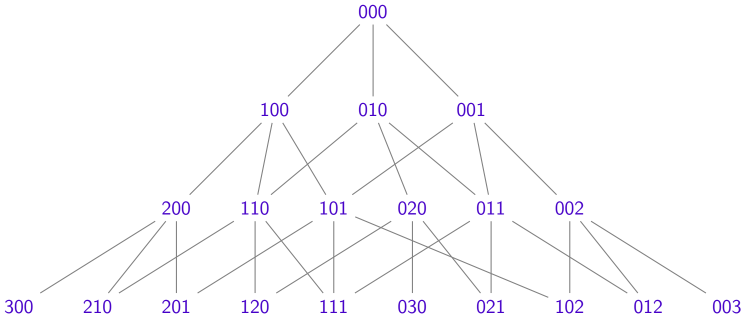

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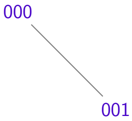

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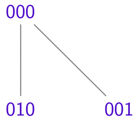

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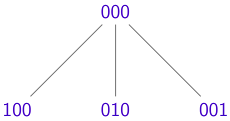

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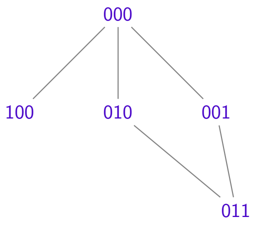

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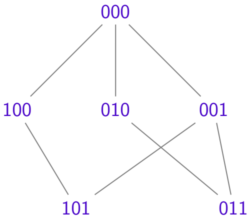

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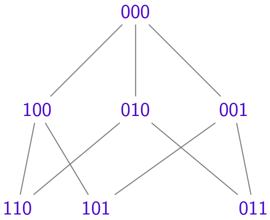

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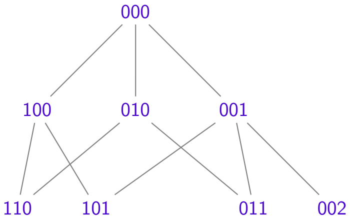

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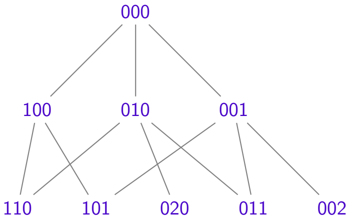

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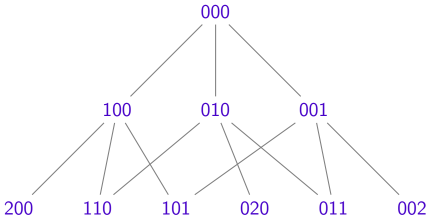

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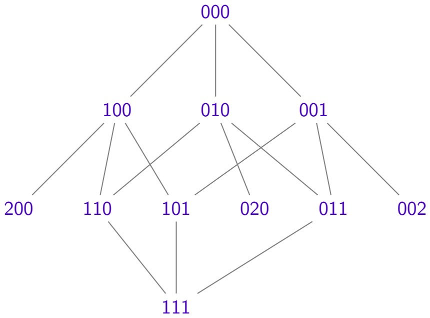

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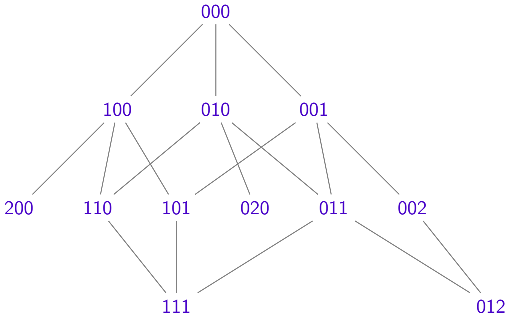

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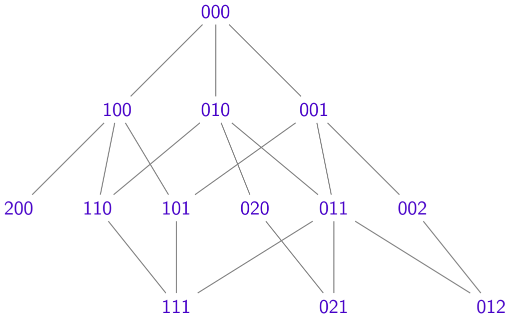

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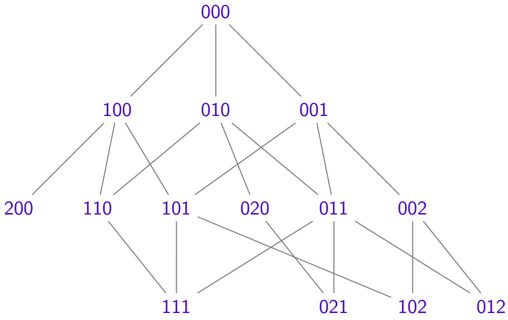

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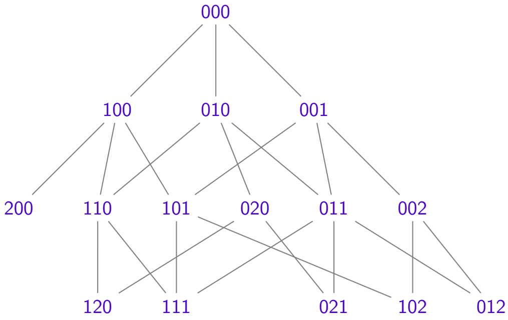

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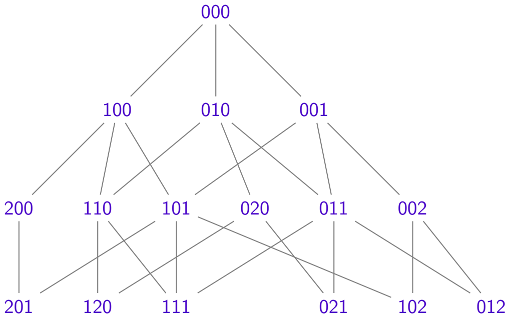

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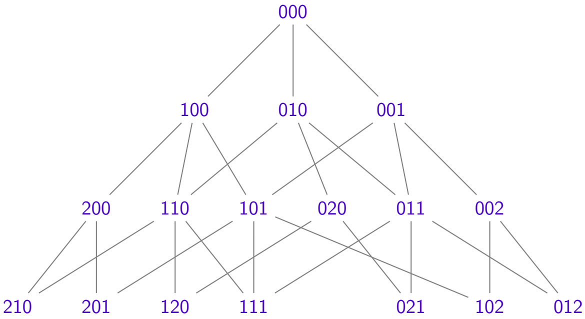

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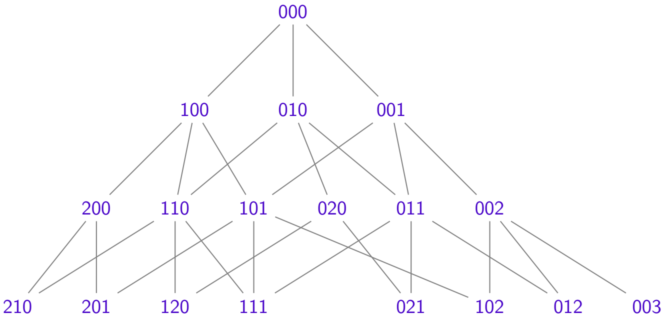

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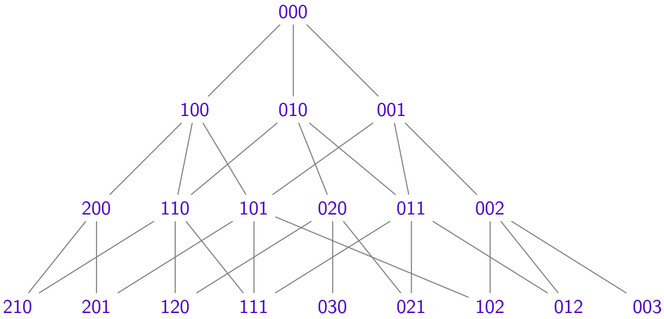

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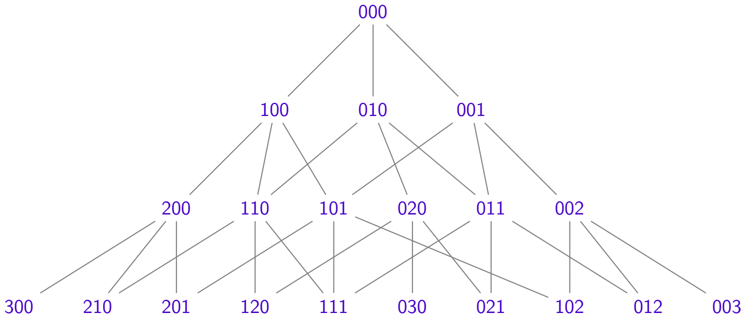

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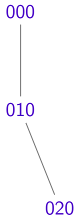

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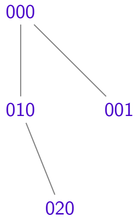

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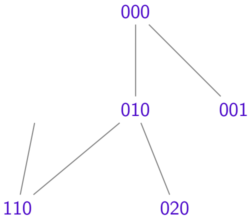

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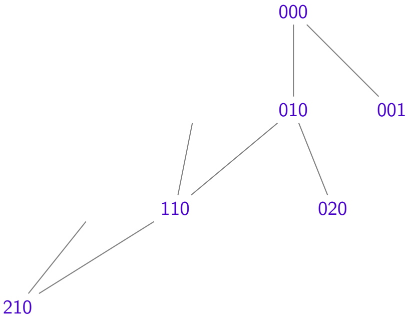

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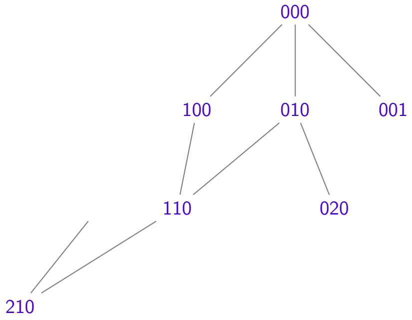

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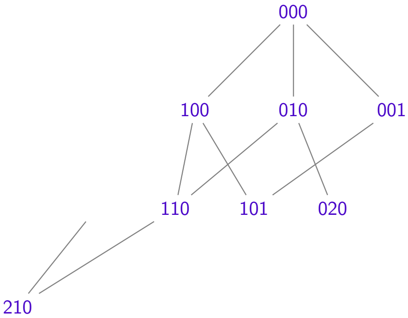

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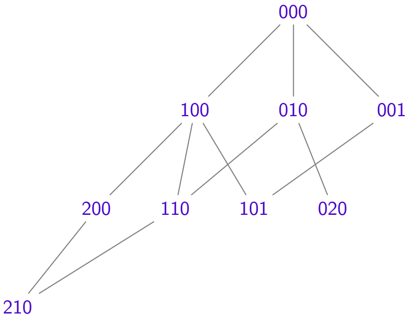

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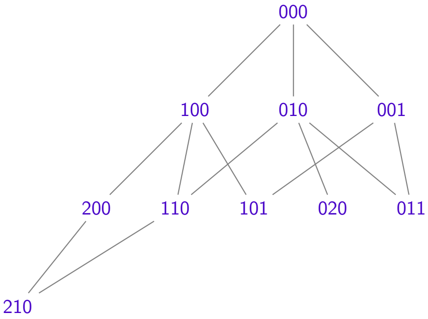

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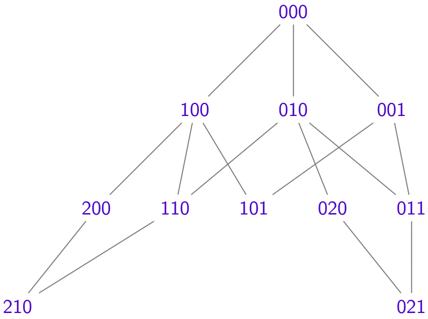

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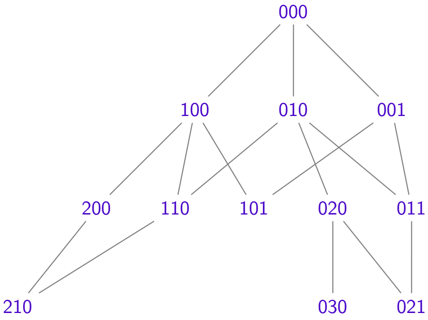

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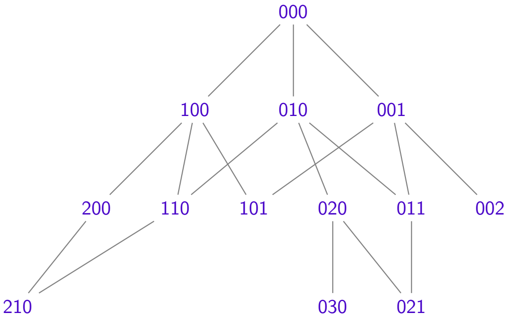

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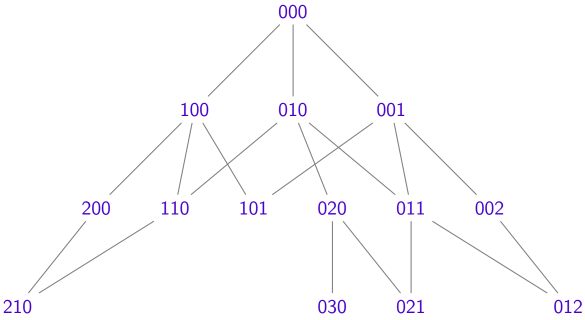

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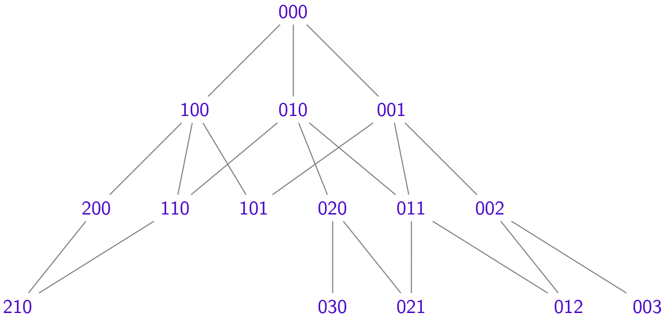

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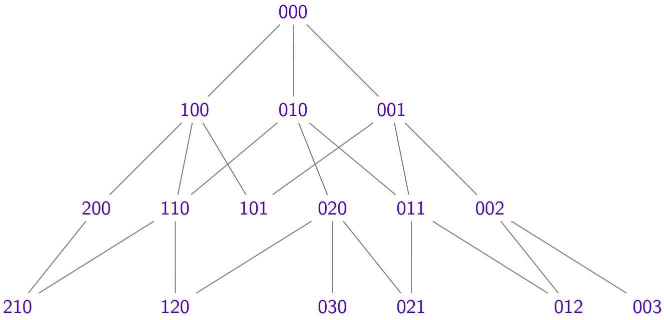

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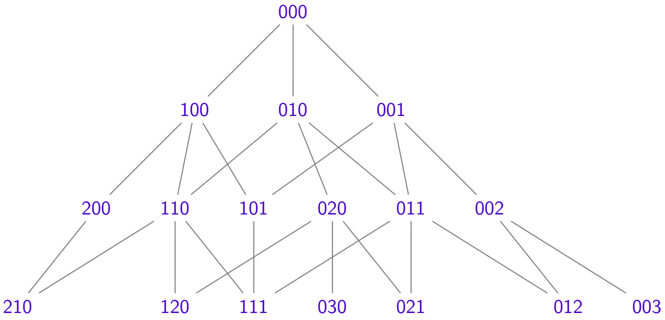

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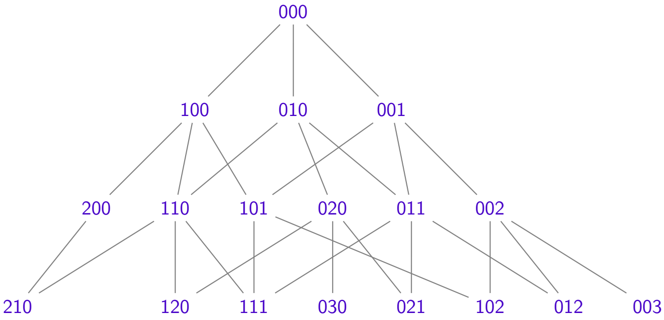

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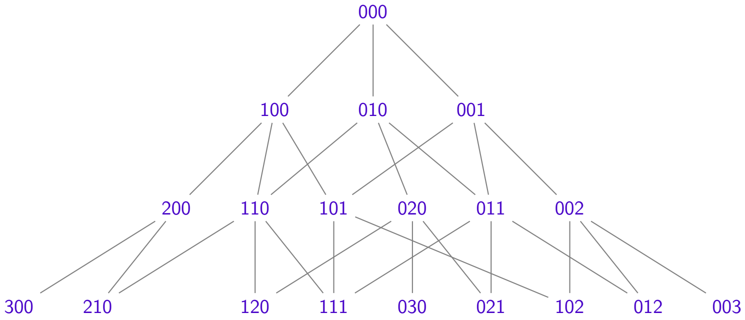

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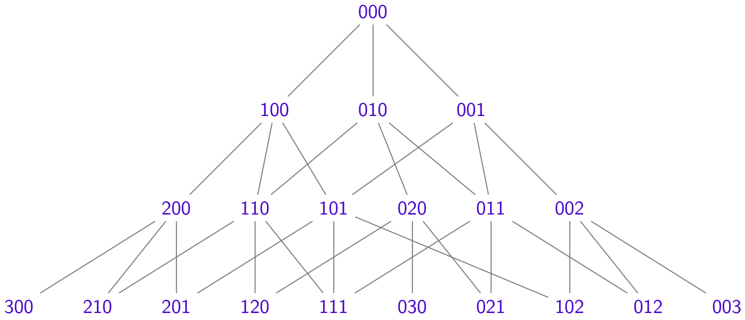

Experiments

Library	#	e	u	id2	id4	lmax	sum	rwlk
TPTP	18627	**7765**	6989	6801	6834	6832	6922	6839
UF	7668	**3243**	3016	2975	2963	2959	3009	2992
UFLIA	10137	**7424**	6024	6018	5897	6001	5980	5994
UFNIA	13509	5715	**7458**	7396	7384	7426	7437	7430

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Library	#	e	enu-vbs	z3
TPTP	18627	**7765**	7330	-
UF	7668	**3243**	3120	2905
UFLIA	10137	**7424**	6188	6912
UFNIA	13509	5715	**7620**	6491

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- We define a number of diverse strategies.
- based on Pareto ordering
- The new strategies diversify the solver.
- Could we somehow learn what is better?
- How to include dependence between variables in the order?

Ge, Y. and de Moura, L. M. (2009).
Complete instantiation for quantified formulas in satisfiabiliby modulo theories.

Reynolds, A., Barbosa, H., and Fontaine, P. (2018).
Revisiting enumerative instantiation.